Beautiful Maths Problems

Materials for Promoting Problem Solving and Experiencing the Beauty of Mathematics

Educating the Educators III

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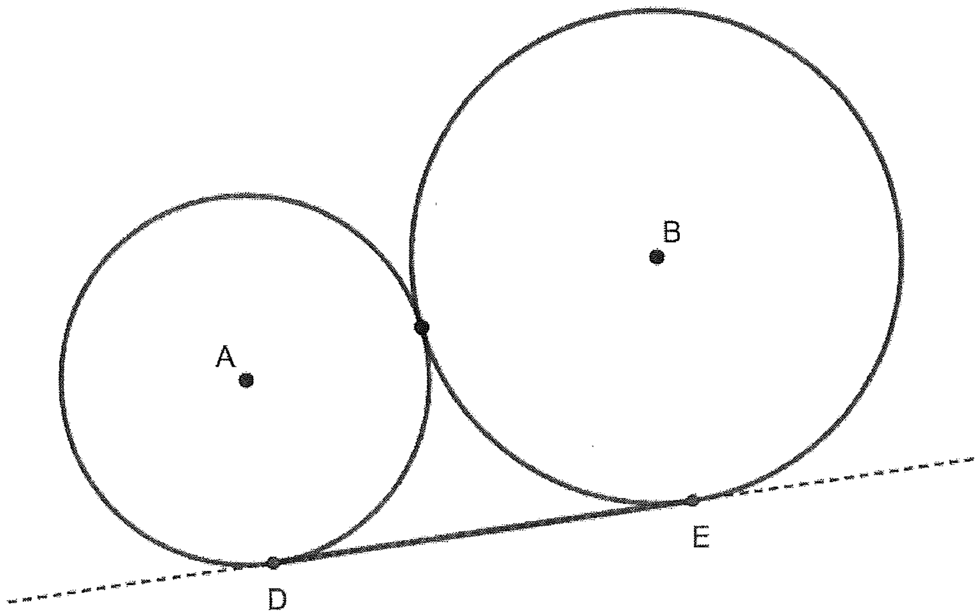
Common External Tangent

The radius of circle A is 5. The radius of circle B is 7. $\overleftrightarrow{DE}$ is a common external tangent. Find distance DE.

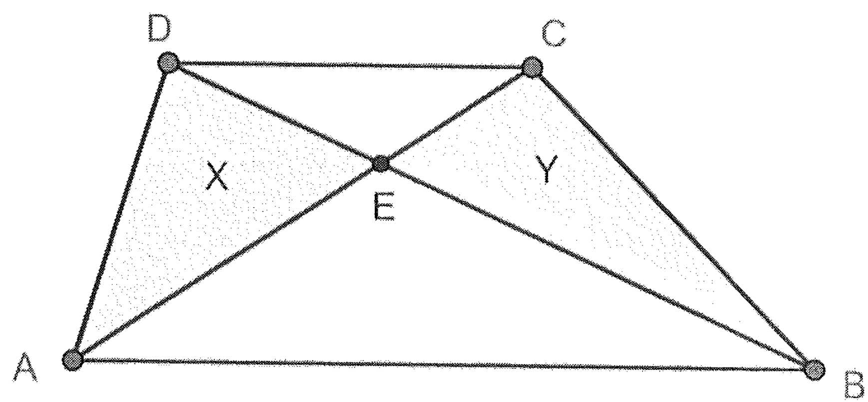


Now Generalize: Use radii of r_1 *and* r_2.

Geometric Mean in a Trapezoid

ABCD is a trapezoid. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at point E

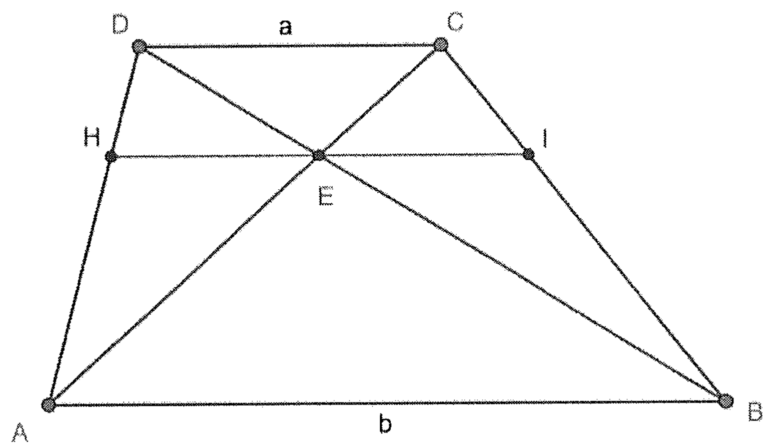
Prove: $Area(\Delta ADE)$ is the geometric mean of $Area(\Delta DCE)$ and $Area(\Delta BAE)$.



Harmonic Mean in a Trapezoid

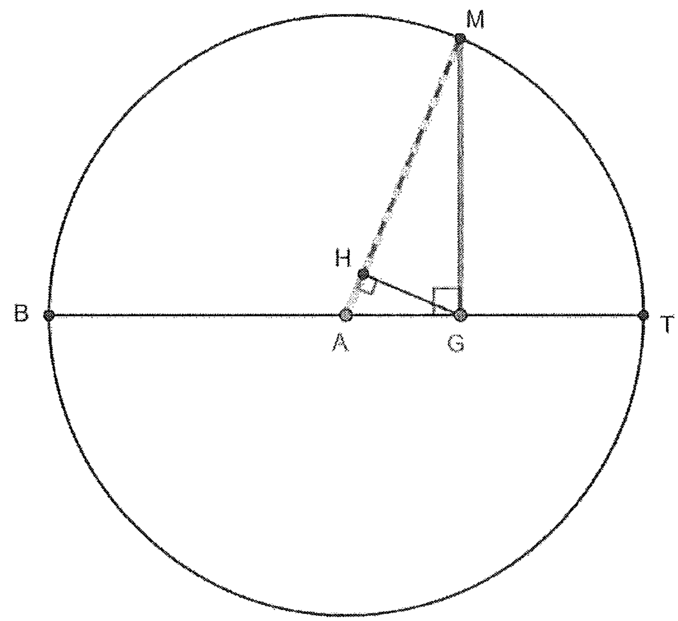
Given: ABCD is a trapezoid. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at point E.
$\overline{CD} \parallel \overline{AB}$ *and* $CD = a$ *and* $AB = b$.
$\overline{HI}$ is parallel to the bases and goes through E.

Prove: $HI = \frac{2ab}{a+b}$



The Three Pythagorean Means in a Circle

Given:



Circle centered at A

$\overline{BT}$ is a diameter

$\overline{GM} \perp \overline{BT}$

$\overline{HG} \perp \overline{AM}$

Show:

$$AM = \frac{GB + GT}{2}$$

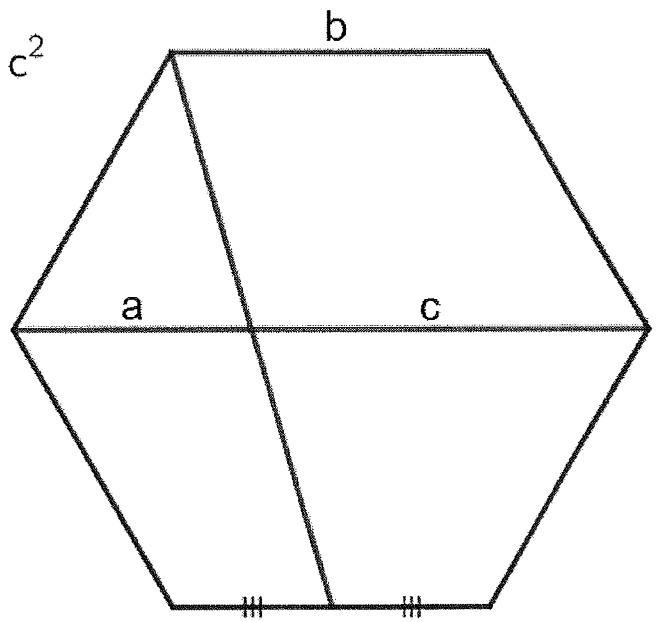
$$HM = \frac{2(GB \cdot GT)}{GB + GT}$$

$$GM = \sqrt{GB \cdot GT}$$

Pythagorean in a Hexagon

Regular hexagon

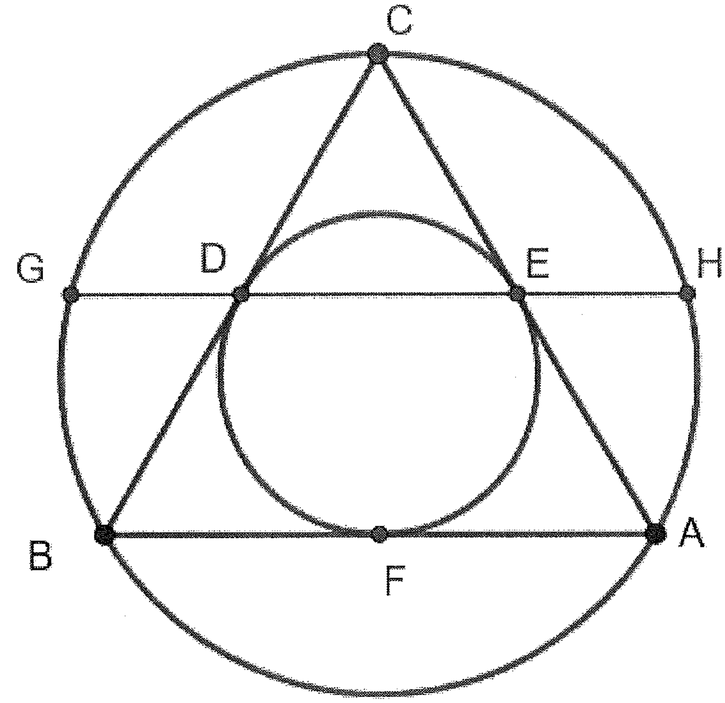
Show : $a^2 + b^2 = c^2$



Golden Ratio Using the Inscribed and Circumscribed Circles

Given: Equilateral triangle and the inscribed and circumscribed circles.
$DE = a$ and $DG = b$.

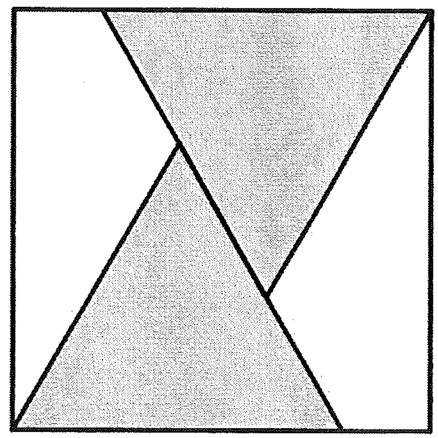
Prove: $\frac{a}{b} = Golden\ Ratio$



Two Equilaterals in a Square

https://twitter.com/HenkReuling/status/1036694353706733568

Two equilateral triangles in a square.
Is more or less than half the square shaded?
(What is the exact ratio?)



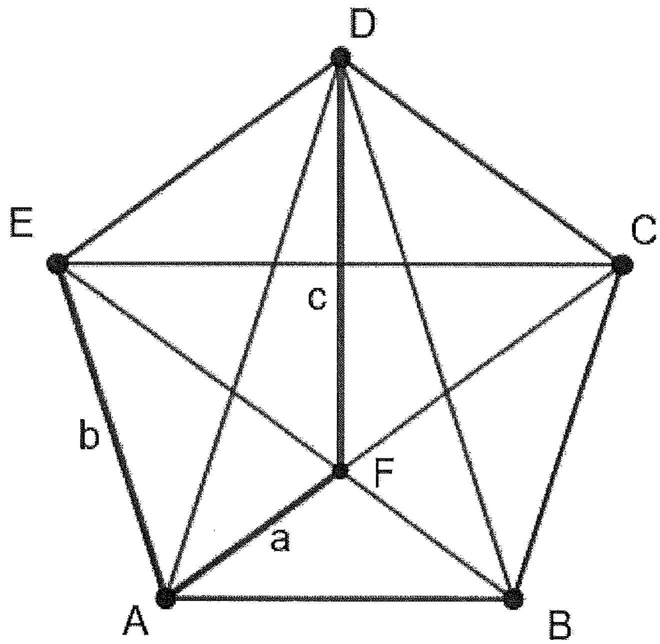
Pythagorean in a Pentagon

Given: Regular pentagon with its diagonals.

$AF = a, \;\; AE = b,$ and $DF = c.$

Prove: $a^2 + b^2 = c^2$

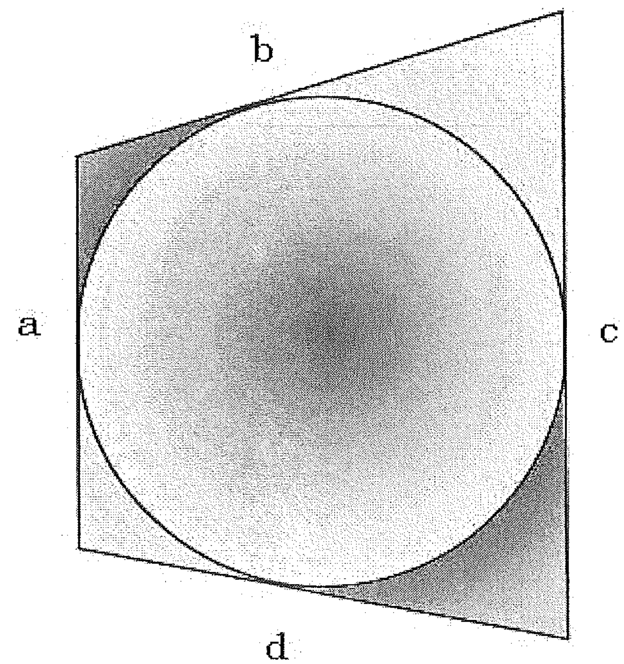
NOT true! oops!



Circumscribe a Quadrilateral

Show the following is true:

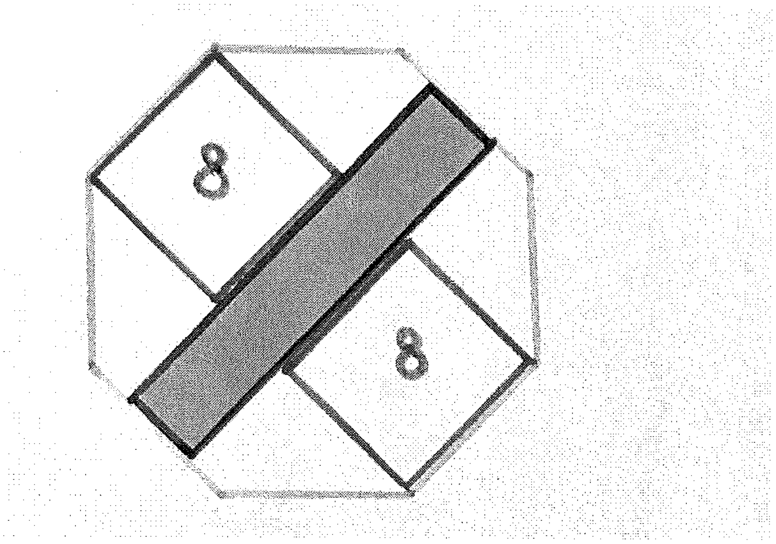
If a quadrilateral circumscribes a circle, then the sums of its opposite sides are equal. That is, in the diagram, $a + c = b + d$.



By Cliff Pickover (@pickover)

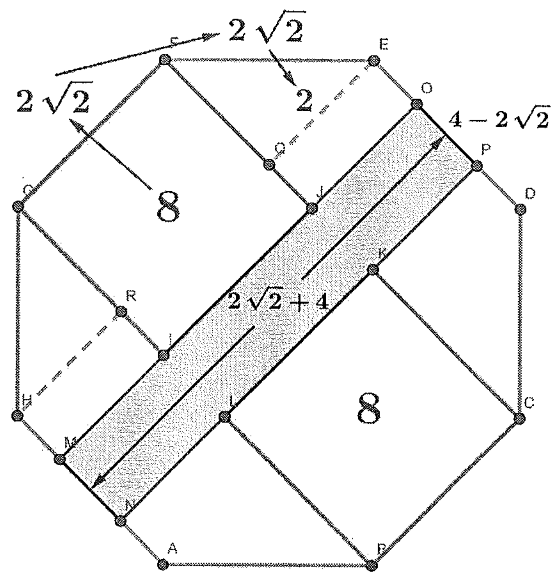
The In-between rectangle in an octagon.

Inside this regular octagon sit two squares of area 8. What's the area of the shaded rectangle?



Source: https://twitter.com/Cshearer41/status/1089227396954419200

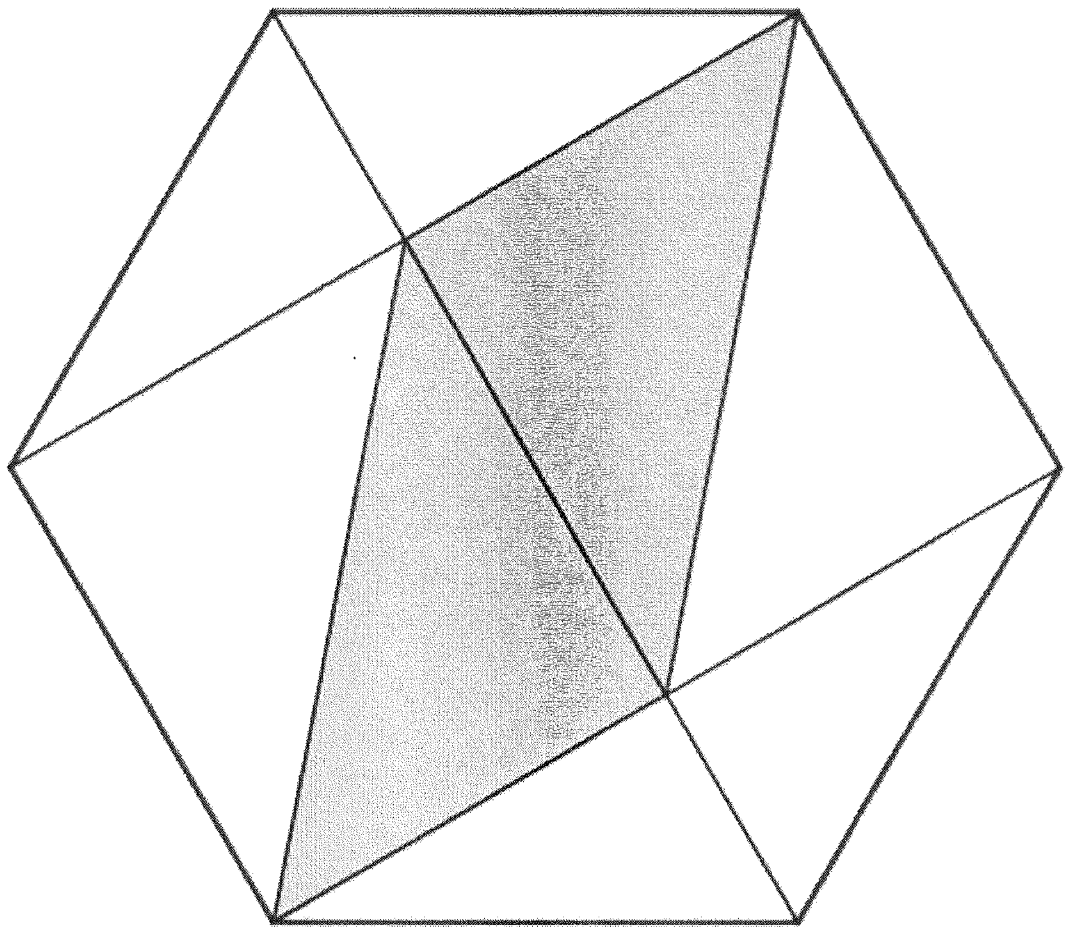
Answer: 8
https://twitter.com/sergiosanz001/status/1089249605349507074



$$\text{Shaded Area}=\left(2\sqrt{2}+4\right)\left(4-2\sqrt{2}\right)=8$$

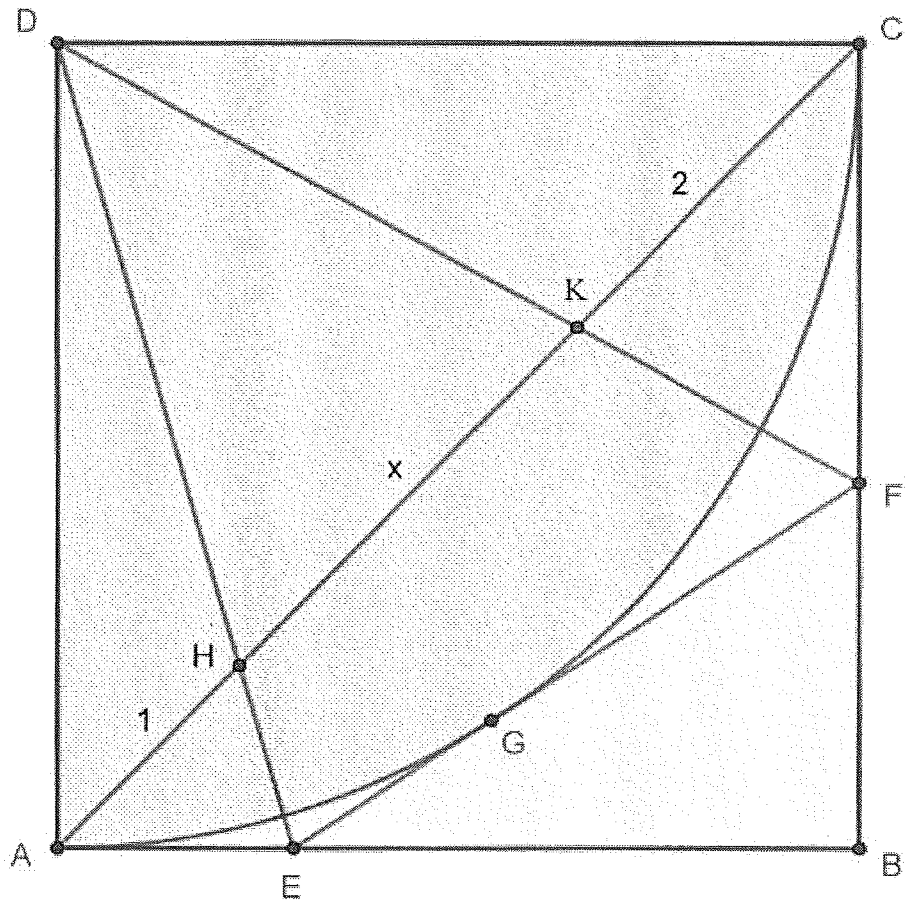
Parallelogram in a Hexagon

In this regular hexagon, three diagonals have been drawn to form this parallelogram. What fraction of the hexagon is shaded?



The Big Finish

A quarter circle is inscribed in a square ABCD with diagonal $\overline{AC}$. Point G is on the circle. The tangent through G meets the square in points E and F.
Draw segments $\overline{DE}$ *and* $\overline{DF}$. This defines H and K.
AH = 1, CK = 2. Find HK.



Source: https://twitter.com/JhuriaMikki/status/1096991539291578370