Handouts

Look For and Make Use of Structure, Symmetry, and Similarity

34th Annual Conference on Teaching Mathematics
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Southern Illinois University – Carbondale

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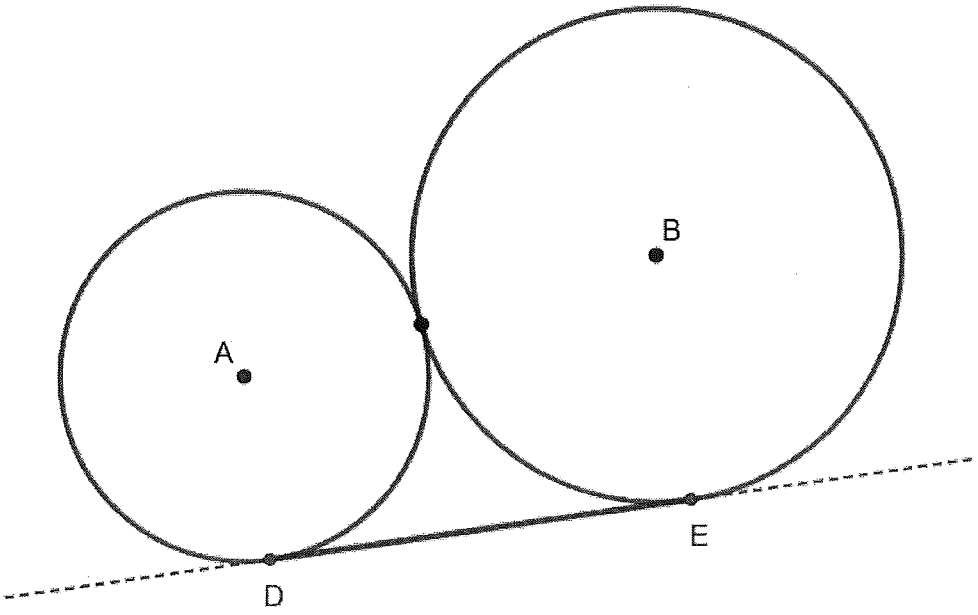
http://faculty.wiu.edu/JR-Olsen/wiu/ - handouts available here electronically.

Common External Tangent

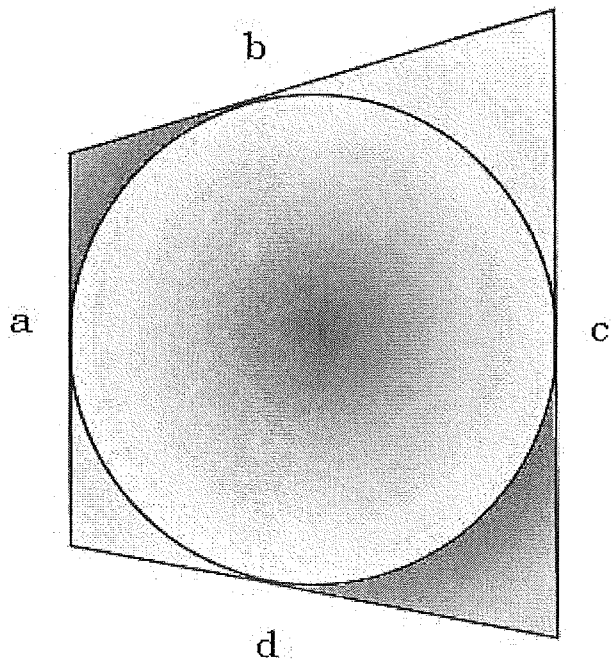
The radius of circle A is 5. The radius of circle B is 7. $\overleftrightarrow{DE}$ is a common external tangent. Find distance DE.



Circumscribe a Quadrilateral

Show the following is true:

If a quadrilateral circumscribes a circle, then the sums of its opposite sides are equal. That is, in the diagram, $a + c = b + d$.



By Cliff Pickover (@pickover)

Jim Olsen, Western Illinois University

A Few Tips for *Looking for and Making Use of Structure*

a) Draw in segments – often to form triangles.

b) Look for symmetry (draw in segments as needed).
- Look for right angles (perpendicular lines).
- Right triangles are nice. Equilateral, 30-60-90°, isosceles, and 45-45-90° triangles are especially nice!
- Look for parallel lines.

c) Draw in radii of circles (*they are all congruent*).

d) If you have a tangent line, a radius drawn to the point of tangency is often very useful ($\perp$).

e) Pay attention to inscribed angles. They have a measure half the intercepted arc.

f) Look for similar triangles and set up proportions.

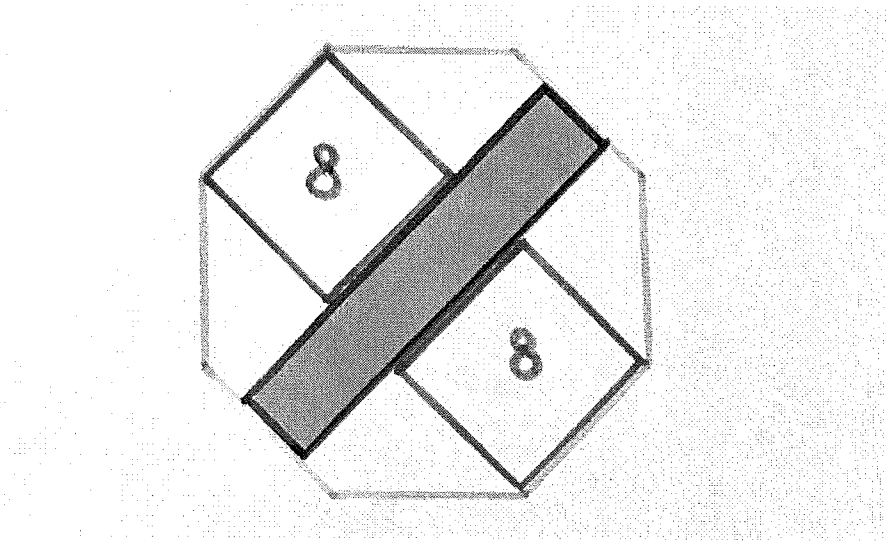
g) Be flexible. If something doesn't work, try something different.

Other Resources of Great Problems/Questions

- http://geometry.drjimo.net/ Structure and Symmetry website (by Jim Olsen)
- https://artofproblemsolving.com/wiki/index.php?title=Main_Page Art of Problem Solving Wiki
  - https://artofproblemsolving.com/wiki/index.php/AMC_Problems_and_Solutions AMC Problems and Solutions (AMC8; AMC10, AMC12)
- Twitter:
  - @Cshearer41
  - @pickover
  - @MathCeyhun
  - @ilarrosac
  - @panlepan
  - @Simon_Gregg
  - @puzzlist
  - @jamestanton
  - @StrummingMom
  - @JhuriaMikki
  - @DrOlsen314 (Jim Olsen)
- Sangaku - *Sacred Mathematics: Japanese Temple Geometry*

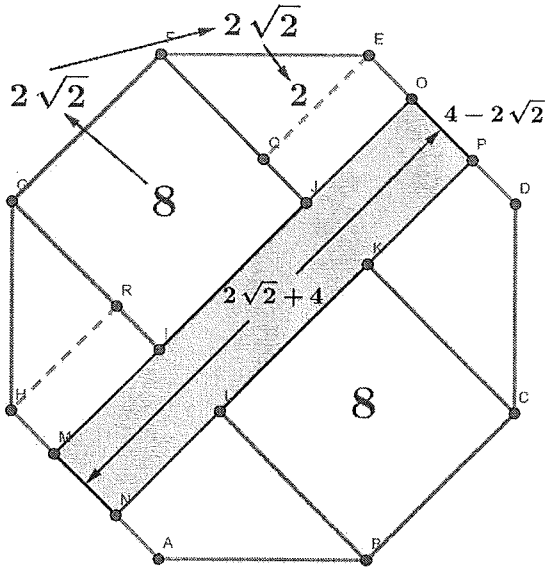
The In-between rectangle in an octagon.

Inside this regular octagon sit two squares of area 8. What's the area of the shaded rectangle?



Source: https://twitter.com/Cshearer41/status/1089227396954419200

Answer: 8
https://twitter.com/sergiosanz001/status/1089249605349507074



Shaded Area= $\left(2\sqrt{2}+4\right)\left(4-2\sqrt{2}\right)=8$

Why These Wonderful Geometry Problems?

By Jim Olsen

Mission Statement:
(My mission is to) Provide a large collection of wonderful geometry problems so that people can develop a connected understanding of mathematics and higher order thinking, see the beauty of mathematics, and experience the joy of mathematical reasoning, as a human experience.

Why – The reasons for these problems and this curriculum:

1. This is exciting. Mathematics has many really cool relationships.
2. Teach problem solving (the world needs problem solvers).
3. Application - We learn x by applying x to solve y.
4. Engaging in problem solving can enhance conceptual understanding and procedural fluency. The *National Mathematics Panel* said, "For all content areas, conceptual understanding, computational fluency, and problem-solving skills are each essential and mutually reinforcing, influencing performance on such varied tasks as estimation, word problems, and computation." (Final report, page 30).
5. These problems usually bring together (show) the interplay (connections) between algebra and geometry (and more, such as proportional reasoning).
6. Help students learn how to look for and express regularity in repeated reasoning.
7. Higher order thinking. Analysis (breaking into parts), synthesis (combining ideas), evaluation (making an informed judgement).
8. Depth of knowledge is gained and demonstrated by applying knowledge to solve problems. Teach critical thinking. To be successful in college and careers, one must be able to think critically. (The world needs people who can think critically).
9. Help students learn how to use appropriate tools strategically.
10. Help students learn how to look for and make use of structure.
11. Creativity aspect - By combining approaches, methods, and techniques one can find creative solutions to problems (the world needs people who can think creatively).
12. Learning pure ***mathematics as a human endeavor***. Furthermore, some can grow to enjoy mathematics (the world needs math majors and math teachers).

Handout for 34th Annual Conference on Teaching Mathematics – SIU-C
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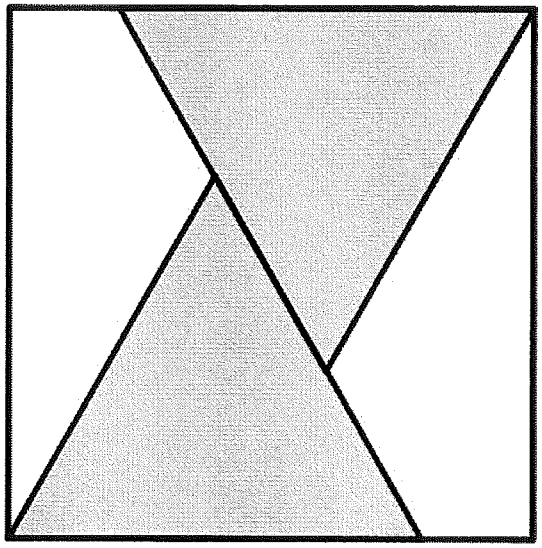
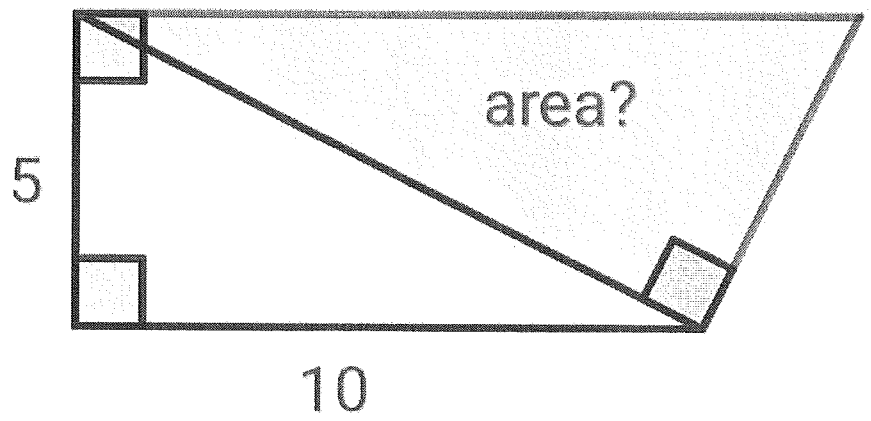
Two Equilaterals in a Square

Two equilateral triangles in a square.
Is more or less than half the square shaded?
(What is the exact ratio?)

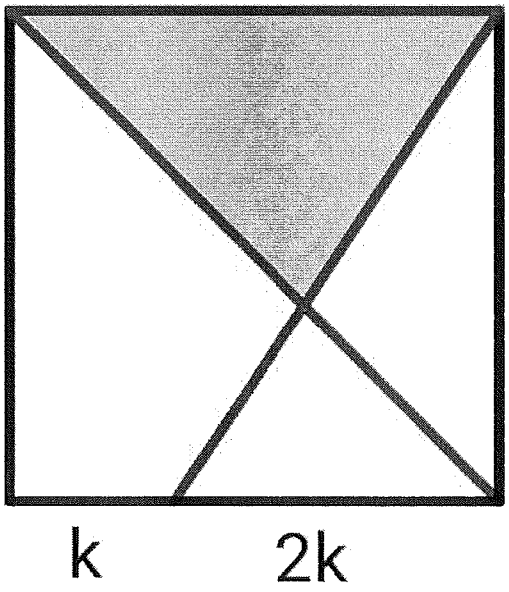


Instant Classic 1



Fractional Triangle in a Square

square

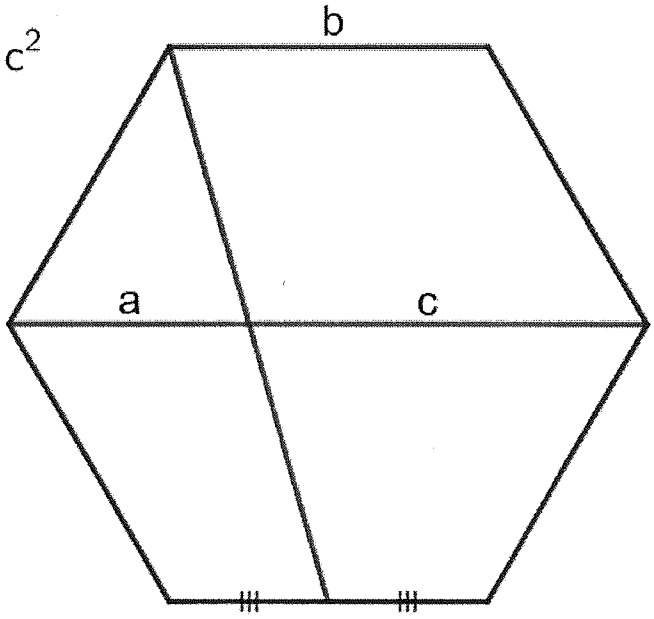


What fraction is shaded?

Pythagorean in a Hexagon

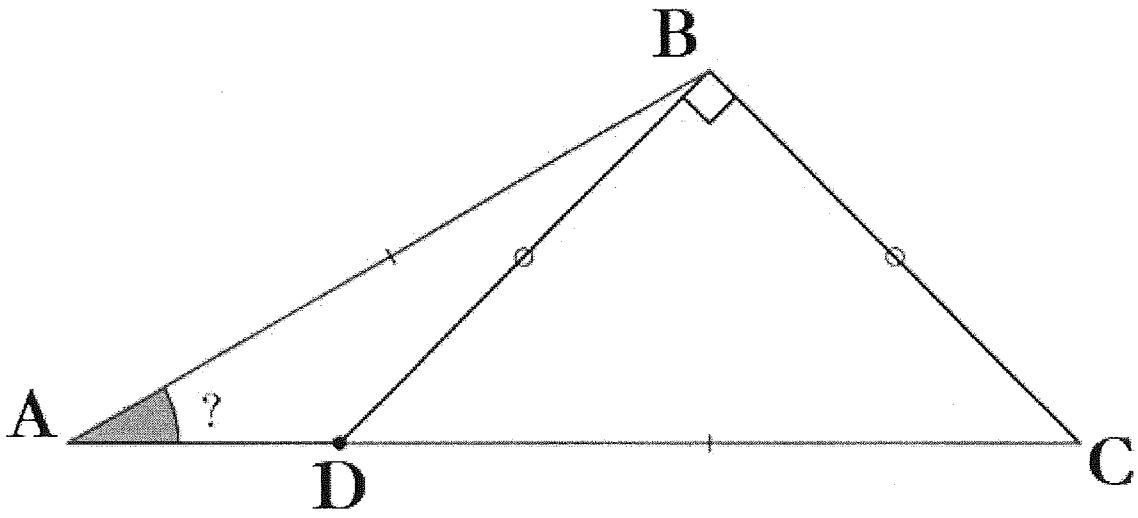
Regular hexagon

Show : $a^2 + b^2 = c^2$



Obtuse Laying on an Isosceles Right Triangle

AB = CD. BD = BC. Find m∠A.

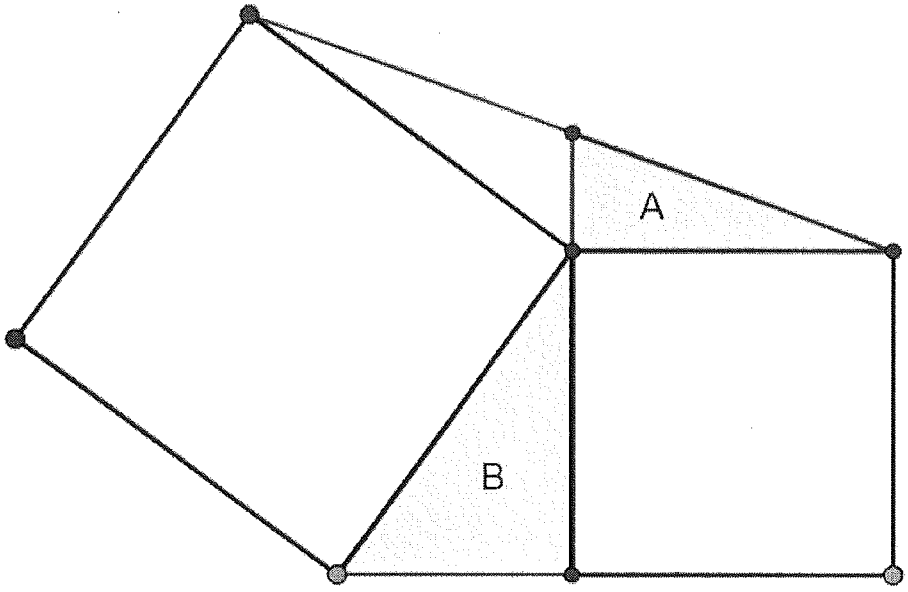


Source: https://twitter.com/puzzlist/status/1093042288886837249

Two Squares and Two Right Triangular Regions

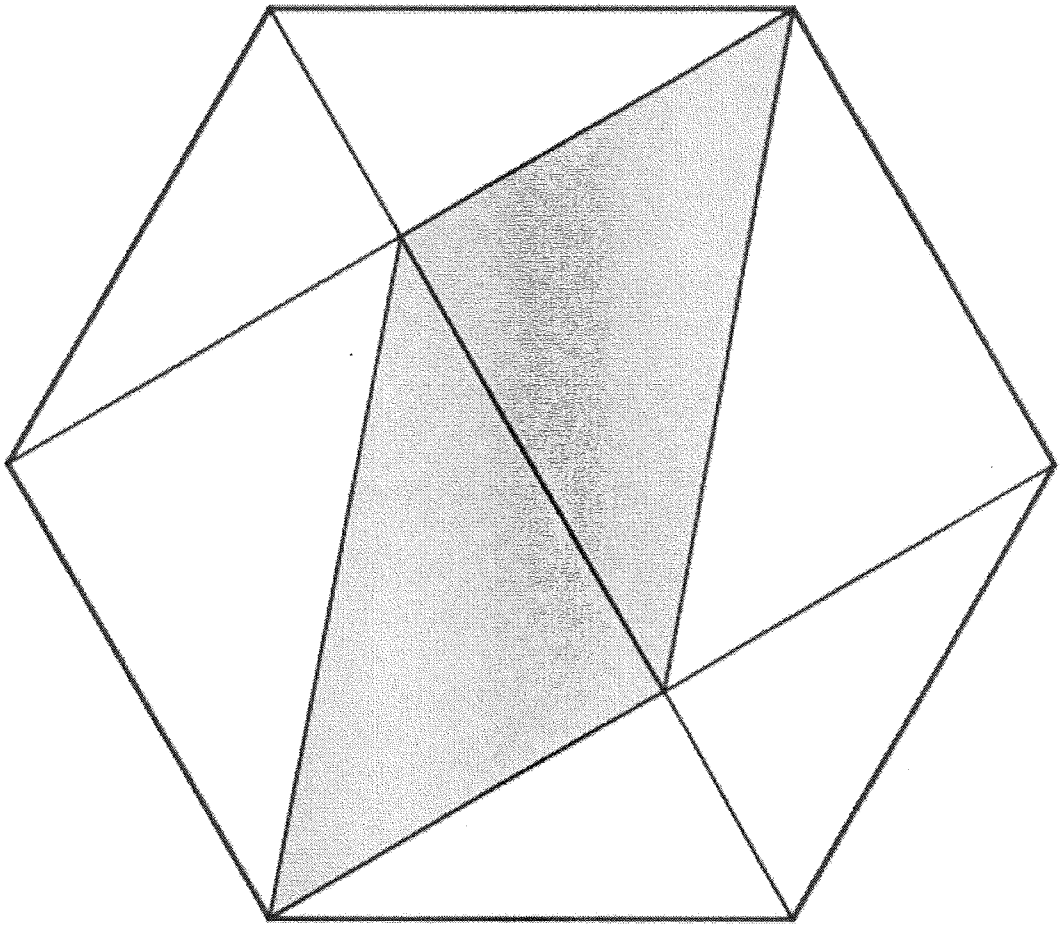
Two squares and two right triangular regions.

Find the ratio of the areas: $\frac{A}{B}$



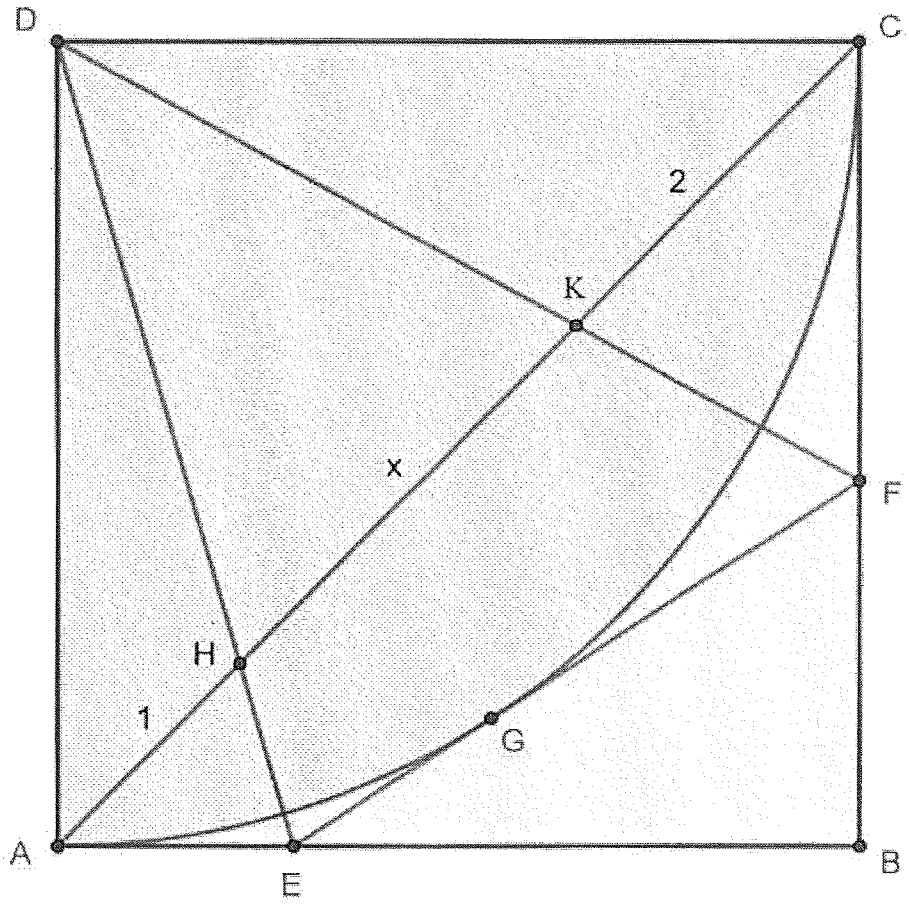
Parallelogram in a Hexagon

In this regular hexagon, three diagonals have been drawn to form this parallelogram. What fraction of the hexagon is shaded?



The Big Finish

A quarter circle is inscribed in a square ABCD with diagonal $\overline{AC}$. Point G is on the circle. The tangent through G meets the square in points E and F.
Draw segments $\overline{DE}$ *and* $\overline{DF}$. This defines H and K.
AH = 1, CK = 2. Find HK.



Source: https://twitter.com/JhuriaMikki/status/1096991539291578370