How to Build a (3D) Model of a Pyramid† Showing the Key Lengths

†Rectangular and *square* pyramid instructions included *[square pyramid information in italics]*

Purpose

The purpose of this model is to help students see the structure of the pyramid and its key lengths.

In particular, this model shows the "essential tetrahedron," whose four faces are *all right triangles*! The model helps students distinguish between the various segments/lengths of pyramid and the essential tetrahedron (whose edges are the three "heights," and half the length, half the width, and radius of the base – see the Key lengths section below). With the ability to visualize the right triangles, students can apply the Pythagorean Theorem (perhaps more than once) to find the necessary lengths to find the surface area and volume of the pyramid.

The model described here has a base which is 4" by 6" *[4" × 4"]* and a 6" *[$\sqrt{41}$"]* height. Other lengths could be used – the process is the same.

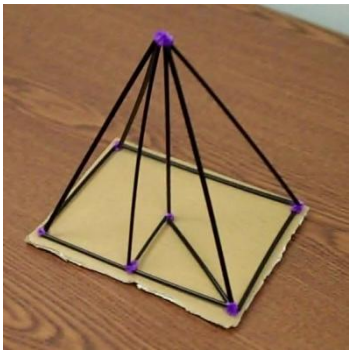
Needed materials and tools

- 4" × 6" card *[4" × 4" card]*
- 5" × 7" cardboard (from a cardboard box, for example – nothing special) *[5" × 5" cardboard]*
- Coffee stirrers or Straws (I use coffee stirrers that are about 7" long)
- Fuzzy sticks ("pipe cleaners" or "chenille stems")
- Ruler
- Scissors
- A compass (or something else, such as a nail, to poke hole in the cardboard)
- Wire cutters (or something to cut the fuzzy sticks)

Key lengths for a rectangular pyramid

The six *-ed lengths make up the *essential tetrahedron*.

- **Width** of the base, and half the width* of the base.
- **Length** of the base, and half the length* of the base.
- **Radius*** of the base (the distance from the center to a vertex).
- **Height*** of the pyramid.
- **Slant height***(s) (the distance from the midpoint of a base edge to the apex). Note: this model only shows one slant heights. For a non-square base, the two slant heights are not congruent.
- **Edge height*** (edge of the pyramid)

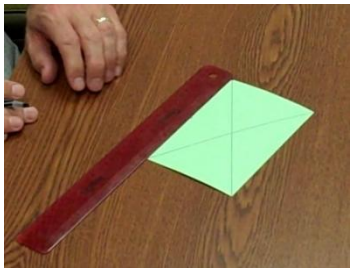


Completed Pyramid

Vocabulary

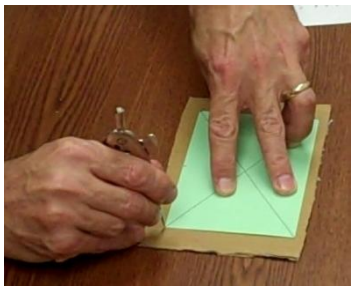
In addition to the terms in the Key lengths section, the following terms are important for communication.

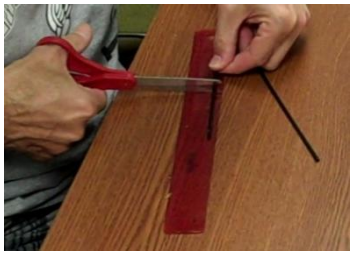
- **Base**
- **Apex** (The endpoint of the height segment which is *not* on the base. If the base of the pyramid is down, then the apex is the top of the pyramid.)
- **Vertex**
- **Right angle**



Step 1 (On Next Page)

Steps for building the model





Preparation

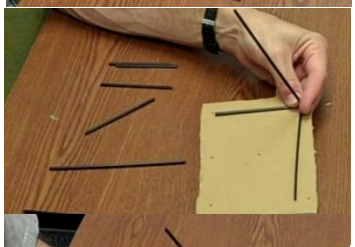
1. Use the 4" × 6" *[4" × 4" card]* card to make the base template. Draw the two diagonals to find the center of the rectangle. Mark the midpoint of one of the 6" sides of the card.
2. Put the 4" × 6" *[4" × 4"]* card (base template) on the cardboard and poke 6 holes in the cardboard: at the four corners, at the center, and at the midpoint of a side.
3. Remove the card and poke the holes all the way through the cardboard (the holes need to just be big enough to poke the fuzzy sticks through).
4. Assemble the fuzzy stick joints (which will be the vertices of the polygons and pyramid). You need: *four* degree-4 joints; *two* degree-5 joints; *one* degree-6 joint. To make the even-degree joints (degree-4 or -6) twist together 2-inch fuzzy sticks (that is, to make degree-4 use two 2-inch fuzzy sticks; to make degree-6 use three 2-inch fuzzy sticks). To make the (odd) degree-5 joints, twist in a 1¼ inch fuzzy stick.
5. Cut the coffee stirrers (or straws). The 13 lengths needed are shown in the table. Note: You need to cut each piece about *one-eighth inch less (shorter)* than the given distance, due to the fuzzy stick joints.

Length	2"	3"	$\sqrt{13}$"	4"	6"	$\sqrt{40}$"	7"
How many	1	2	1	2	2	1	4
How many	*2":3*	$\sqrt{8}$*":1*	*4":3*	$\sqrt{41}$*":1*	$\sqrt{45}$*":1*	*7":4*	

(Examples: for the 2" piece, cut it about $1\frac{1}{8}$" long; the 3" piece, cut it about $2\frac{7}{8}$" long; for $\sqrt{13}$" use 3½")

Assembly

6. Slide the coffee stirrers on to the joints. The joints at the base will each have an *extra* fuzzy stick branch without a coffee stirrer (see the next step). (Pictured on the next page)
7. Poke the extra fuzzy stick branches through the holes in the cardboard base. (Pictured on the next page)
8. If the model "binds," slightly shorten (trim) the coffee stirrers. (See Hint a below.)



Helpful hints

a. It is better that the coffee stirrers are a little too short than a little too long.
b. For fuzzy sticks, a little too long is better than a little too short.
c. The cardboard base can be any size as long as it is bigger than 4" × 6".
d. (Optional) Tape "flags" on the segments (fuzzy sticks) to label the names or lengths.

Final note

- This model can be disassembled, put in a zip-lock bag, and stored for future use.

Video: http://youtu.be/hLY1niOHQ7o

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