Abstract: There are many dynamical systems that can be modeled with systems of linear differential equations, and hence can be written in a matrix equation form. A vibrating system of masses and spring is one such dynamical system. The eigenvalues of the coefficient matrices are the natural frequencies of the system. An inverse problem explores if there exists a system of masses and springs for a given set of natural frequencies. Here is a question:

Question 1. Find a 4x4 real symmetric matrix with eigenvalues -2, -1, 1, and 2.
There is a trivial answer to that question. However, here is a slightly different question:

Question 2. Find a 4x4 real symmetric matrix whose eigenvalues are -2, -1, 1, and 2, so that none of the entries of the matrix are zero.

The Implicit Function Theorem (IFT) implies that if you have a 'nice' function, small perturbations of that function are also 'nice'. We will use the IFT to modify the trivial solution of Question 1 to a solution for Question 2, and explore more general implications.

Question 3 (Bonus question). Find a 4x4 real skew-symmetric matrix with eigenvalues -2i, -i, i, and 2i.)