Tight and Efficient Geodesics in the Curve Complex

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Abstract: The curve complex of a surface is a simplicial complex with vertices denoting isotopy classes of closed loops (henceforth referred to as curves) on the surface. Two vertices are joined by an edge if their corresponding curves are disjoint, and the distance between two curves on the surface is then defined to be the length of the geodesic (shortest) edge path connecting the corresponding vertices.

In general, between any two fixed vertices there is an infinite set of geodesics, but two finite subsets of geodesics have been identified, namely tight geodesics by Masur and Minsky and efficient geodesics by Birman, Margalit and Menasco. The latter group of researchers have shown that these two finite subsets do not precisely coincide; specifically, via a finite list of examples they show there exist geodesics that are (1) efficient and tight, (2) tight but not efficient, and also (3) efficient but not tight. We show that, in fact, there are infinitely many examples of geodesics satisfying these three conditions, and prove a series of propositions whereby these infinite families can be generated.

Mingzhu Wang has studied under the supervision of Dr. Douglas LaFountain.