AN OPTIMAL UPPER BOUND ON REGULARITY OF POWERS OF EDGE IDEALS

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Abstract: The main themes of this talk are the Castelnuovo-Mumford regularity and the edge ideals of finite simple graphs. The Castelnuovo-Mumford regularity (or simply regularity) of a graded module over a graded ring is a measure of complexity in the sense that an ideal generated by higher degree polynomials is more complex. Homogeneous ideals in polynomial rings with low regularities are known to have simple minimal free resolutions. This motivates mathematicians to find classes of homogeneous ideals with regularities bounded by certain values. Monomial ideals are examples of homogeneous ideals that come with lots of combinatorial data and it becomes interesting to interpret the algebraic properties in terms of the combinatorial properties.

Let $R = K[x_1, \ldots, x_n]$ be a polynomial ring over a field $K$ and $G$ be a graph on the vertex set $V = \{x_1, \ldots, x_n\}$ with edge set $E = \{e_1, \ldots, e_q\}$. The edge ideal of $G$ is defined as follows over the polynomial ring $R$:

$$I(G) = (x_ix_j : \{i, j\} \in E).$$

Edge ideals were first defined in 1990. It is used to make a link between combinatorial properties of a graph and algebraic properties of a quadratic squarefree monomial ideal.

Here in this talk we first give an upper bound for the regularity of powers of edge ideals and then we partially answer this question.