A Nearly Linear Phenomenon, and Exponentiation & Computability

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Abstract

The year 2012 is Alan Turing’s centenary; in 1936, he began a paper with: ‘The “computable: numbers may be described briefly as the real numbers whose expressions as a decimal are calculable by finite means.’

In this talk, we go over examples of familiar computable reals in the works of Taylor, Newton, Machin (Taylor’s co-advisor), and Euler, including discussions of approximating Pi and the golden ratio, and describe current related research and our findings.

We use Euler’s Machin-like formula and produce algorithms to isolate, via spectrum and binary expansion, a certain constant that is involved. Our constant will be irrational by a recent result due to Jahnel.

But there is a hierarchy of primitive recursive reals. Thus there are interesting issues when one restricts primitive recursiveness to around the level E^2 in the Grzegorczyk hierarchy. At that level or below, exponentiation is not available. So the proof that spectrum-E^n reals for n>=3 are base-E^n does not work for n=2. Some have conjectured the existence of a Cauchy-E^2 real which is not spectrum-E^2. It would be nice to know whether the spectrum of the constant arccot(2)/Pi that we will come across in this talk has an E^2 algorithm. Some suspect this number is not automatic; its transcendence, too, remains open.