Title: Computable Reductions Between Mathematical Theorems

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Abstract: Mathematicians often speak informally about how one theorem reduces to another. We have an intuitive sense of what this means, but it is nontrivial to turn this into a precise definition. After all, given any two mathematical theorems P and Q, we could "use" Q to prove P by simply proving P directly while ignoring Q, or by applying it in some inessential way.

One way to get around this is to work only over weak axioms, and this leads to the very successful framework of reverse mathematics. This approach has developed into a robust theory, but it requires significant background in syntactic logic and is in many ways too coarse. We will discuss a new approach that says P reduces to Q if there exists a uniformly computable transformation from instances of P into instances of Q, and another backward uniformly computable transformation from solutions to an instance of Q to solutions of the corresponding instance of P. This approach leads to finer distinctions between mathematical theorems and arguably a more natural mathematical notion of reduction due to its avoidance of formal systems.

This work is joint with Francois Dorais, Damir Dzhafarov, Jeff Hirst, and Paul Shafer.