Continuous Logic and the Model Theory of Metric Structures

Professor C. Ward Henson
University of Illinois

Abstract: Continuous first-order logic is a real-valued generalization of the usual first-order logic. Its propositional fragment corresponds to the logic of Łukasiewicz, which was introduced in about 1930. With quantifiers corresponding to the operations of sup and inf on the interval [0,1], this logic was studied in the 1960s, and then dropped without being substantially developed and without model theoretic applications. Recently it has re-emerged as the appropriate logic for the model theory of metric structures. Its theoretical features have been developed rapidly and its applications are being actively pursued.

Metric structures arise in all areas of mathematics, especially in analysis, probability, and geometry. Examples include: measure algebras; balls in Banach spaces, -lattices, -algebras, etc; in C*-algebras and in asymptotic cones of finitely generated groups; and metric spaces themselves. There is a natural ultraproduct construction for metric structures, which has been extensively used in areas such as Banach space geometry, tracial von Neumann algebras, and geometric group theory. This construction fits beautifully with continuous logic.

In this talk we will briefly present the syntax and semantics of continuous logic for metric structures, indicate some of its key theoretical features, and show a few of its recent application areas. Throughout, there will be an emphasis on specific metric structures of general mathematical interest and the methods used by model theorists to study them. Examples discussed will include such structures as probability algebras, asymptotic cones of finitely generated groups, Urysohn’s metric space and Gurarij’s Banach space.