From graph theory to set theory and back

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Abstract: Descriptive combinatorics is a fascinating area at the interface of combinatorics and descriptive set theory, with deep connections to other subjects, such as ergodic theory. The goal is to understand the behavior of classical combinatorial concepts under additional "regularity" assumptions, i.e., when they are required to be "definable" or at least "well-behaved" in a suitable measure-theoretic or topological sense. In this talk, we will focus on coloring problems—assigning a "color" to each element of a given structure in a way that fulfills a specified set of constraints. For instance, one might want to color the vertices of a graph so that adjacent vertices receive different colors. Standard compactness arguments usually reduce the general situation to the case when the underlying structure is finite. However, as compactness is inherently dependent upon the Axiom of Choice, this approach is "non-constructive" and says nothing about the "definability" of the resulting coloring. But maybe not all is lost: It is conceivable that the existence of a "well-behaved" coloring is equivalent to a stronger assertion in the finite case, perhaps that a finite coloring can be found via an algorithm of a certain form. After a general introduction to descriptive combinatorics, I will outline a result that confirms this suspicion in some cases (when the underlying notion of "well-behavedness" is Baire measurability).