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Gaussian Laser Beams

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1. Introduction

Gaussian laser beams are the simplest and often the most desirable type of beam provided by a laser source, for reasons which also become obvious in the course of this experiment. This experiment seeks to investigate the intensity distribution of a laser beam in three-dimensional space. We will measure this intensity distribution by a combination of various techniques which will be introduced by the instructor. We will learn the use of WINTV for recording pictures from the CCD camera, and ImageJ for measuring and analyzing these pictures taken.

The main experiment will involve first calibrating the CCD camera, and then measure the gamma parameter of the camera; measure the laser beam size before and after a lens is used to focus the beam.

2. Theory

In optics, a Gaussian beam is a beam of electromagnetic radiation whose transverse electric field and intensity (irradiance) distributions are well approximated by Gaussian functions. The strength of the electric field of the simplest mode of a laser is radically symmetric and given by the Gaussian profile given in Eq. 1.0.

$$E(r) = E_0 e^{-(r/\omega)^2} \quad , \qquad 1.0$$

where **r** is the radial distance from the beam axis and ω is the characteristic beam radius. Since the intensity I is proportional to E^2, the radial dependence of the intensity is given by

$$I(r) = I_0 e^{-(r/\omega)^2}. \qquad 2.0$$

Such a Gaussian beam profile is illustrated in Fig. 1.0, where the distance from the beam axis is measured in units of the beam radius ω.

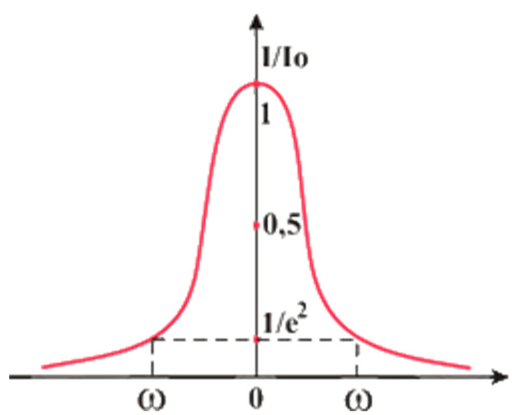

Fig.1.0. Gaussian intensity profile of a laser beam [Source: Google Images]

As the laser beam propagates its diameter may change, the beam may either converge or diverge. The minimal size spot, where the beam has a radius ω_0 is called the beam waist. In general, the beam radius is

$$\omega(z) = \sqrt{1 + (\frac{Z}{Z_0})^2} \quad , \qquad 3.0$$

where $Z_0 = (\pi\omega_0{}^2/\lambda)$ and is called the Rayleigh length. In the far field, (*i.e.* at a distance z >>z_0 from the minimum at z = 0, the beam radius is given by

$$\omega(z) = \frac{\lambda z}{(\pi\omega_0)} \,. \qquad 4.0$$

A measurement of the beam radius as function of z therefore allows the determination of ω_0 for known wavelength or, if the waist can be measured, determination of the wavelength.

3. Experimental Details

3.1 Equipment Utilized

All equipment used is according to standard and as recommended by the instructor. These included: A 640x480 pixel CCD camera (see Fig. 3.1), 2.54mm flat washer ring, ImageJ software for measuring and analyzing pictures taken, TV monitor, WinTV7 for taking snapshots from the CCD camera, lens tube, black cloth, ND filters, Print scale, and a He-Ne Laser, clamps for holding the optical apparatus in place. A flash light came in handy during the times when surrounding lighting will be turned off to achieve better results.

Fig. 3.1 The CCD camera (Source: AMO Lab at Dept. of Physics, Western Illinois University)

3.2 Procedure

We first examined the basic parts of the experimental setup, learning through the instructor, functions of these parts. Ample time was spent on discussing the functioning of the CCD camera and how this is interfaced with the TV monitor. We also took a basic tour of the ImageJ software looking at the various methods for analyzing the pictures. ImageJ has a detailed manual which can be used for learning the functioning of the software. However, we only needed a few steps which involve opening (on the **File** menu, choose **open**, and browse to the folder which contains the file to be uploaded) the photo, and selecting, using the select tool, a rectangular area in the image which is to be projected. Next step is to click **plot profile** option on the **Analyze** menu (see Fig. 4.10 in section 4.0 for a sample plot produced). To change the axis of measurement, say to 'vertical', click **options** -> **plot profile** options on the **Edit** menu, and then check the **vertical profile** option in the pop-up box. This was a lot to remember in the first few tries, but it soon becomes easy after a few steps. We assigned Stewart Ferrell to take the snap shots and generate the plot profiles so that other members can concentrate on the remaining tasks.

After this initial introduction, we then went on to calibrate the CCD camera. Our goal at this point is to take a picture of the 2.54 mm ring washer and use the analyzed photo to calibrate the camera. However, the photo was already taken at the time of this experiment, so we just went ahead with the analysis. Fig. 3.20 shows a picture of the flat ring washer that was measured. The pixel sizes for both the x and y axes were calibrated. Results are shown in section 4.0.

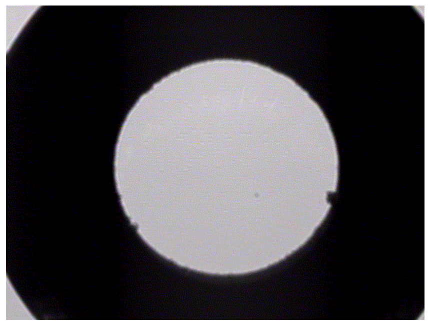

Fig. 3.20. A Picture of the 2.54 mm flat ring washer

Measuring the gamma parameter of the CCD Camera

First, we adjusted the lens of the CCD camera until a fine image of the surrounding environment or object in focus can be seen on the TV monitor. The camera was then focused on the print scale, and adjusted again. In order to obtain an even clear image for better results, we attached a lens tube to the camera and then covered both the print scale and camera with a black cloth. This essentially ensures that no ambient or reflected light is recorded by the camera. A picture of the print scale is then taken as seen in Fig. 3.32 below.

Fig. 3.2 A picture of the print scale.

Again, using the procedures described earlier, ImageJ was used analyze and then to measure the relative intensity. In this case, we selected a rectangular area in each sector of the print scale and analyzed these sectors separately to obtain a series of results (see section 4.0 for these).

The gamma parameter was then found using $I_{out} = A(I_{in})^{\gamma}$. The slope of the plot of $I_{out} - I_{in}$ in logarithmic scale as in Eq. 3.1 below, thus gives the gamma factor.

$$\text{Log I}_{\text{out}} = \text{Log A} + \gamma\text{Log I}_{\text{in}} \qquad 3.1$$

In the above, I_{in} is the obtained from the transmutivity of the neutral density filter sectors of the print scale, while I_{out} is the relative intensity recorded from analyzing the picture.

Measuring the laser beam size before the lens

At this point we experimented with a couple of the ND filters which were used to attenuate the intensity of the camera before the laser hit the camera. First we tried with filter with ND=5 and worked all the way down to about ND=3.0 for the most convenient results. An image of the laser beam was taken as seen in Fig 3.23. and this was again analyzed accordingly. We took a measurement of plot analysis both along the x-axis and y-axis. Then a Gaussian curve fitting was done in ImageJ (see Fig. 3.24). As noted in the laboratory instructions, the parameter 2d in the curve fitting equation shown on the plot is the beam radius. Results are shown in section 4.0

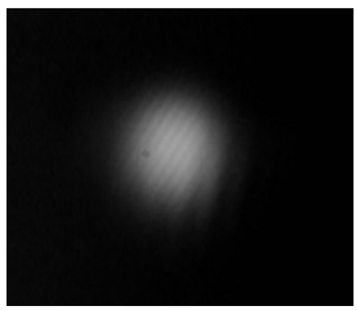

Fig.3.22. Laser beam picture before lens

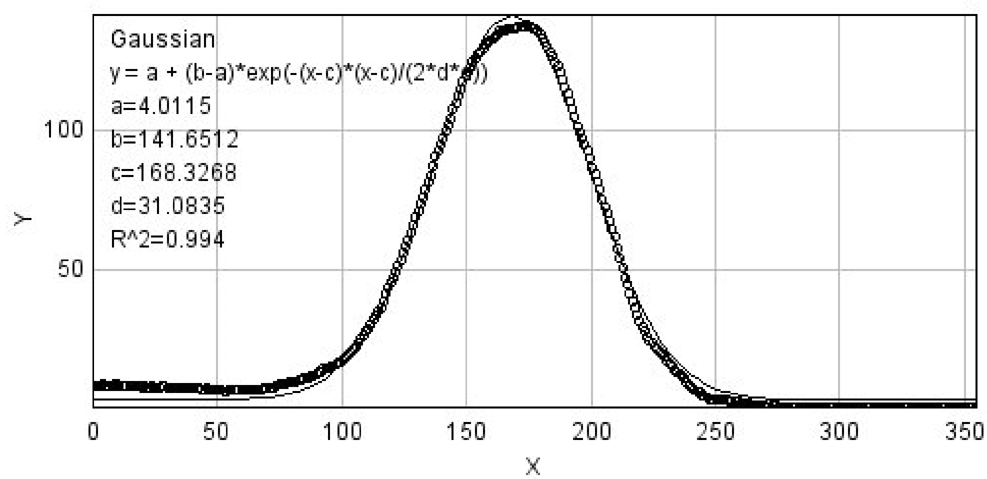

Fig. 3.2.3. Gaussian fit of the projection of laser beam before lens (x-direction)

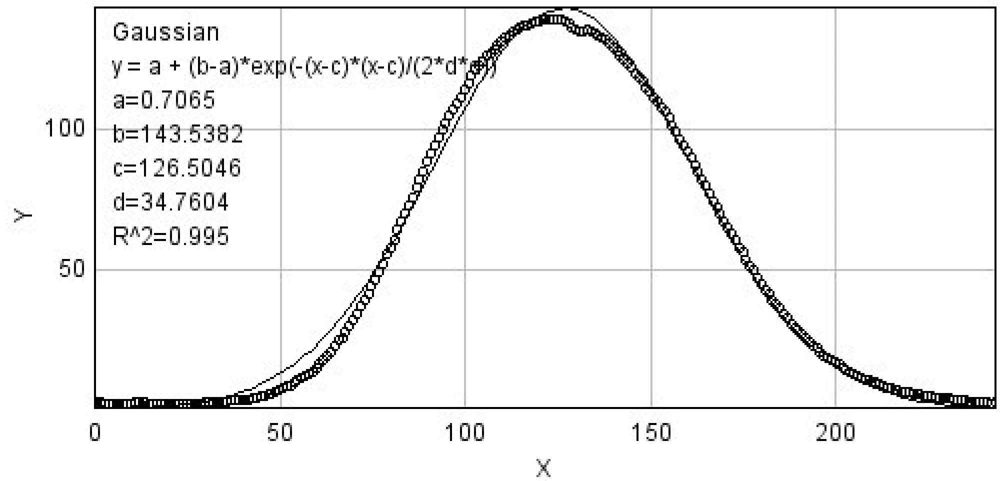

Fig. 3.24. Gaussian fit of the projection of laser beam before lens (y-direction)

Measuring the laser beam size of the focused laser beam

We removed the CCD camera and placed a 69mm lens. Then the camera was again scanned along the z direction, taking the pictures and analyzing as before. In all, 16 pictures was taken and analyzed. The results section shows these results and the beam distribution curve. Table 4.10 gives the results obtained.

4. Results and Discussions

Fig. 4.10 and Fig. 4.11 show the projection of the washer along the x and y axis respectively.

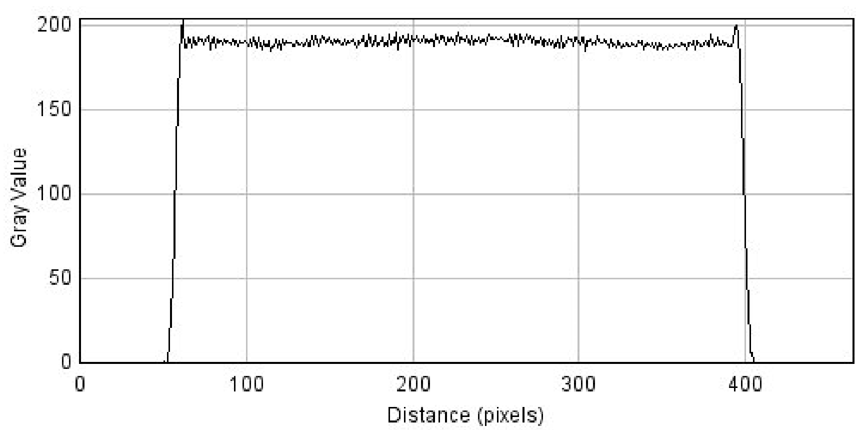

Fig. 4.10 the projection of washer along x axis

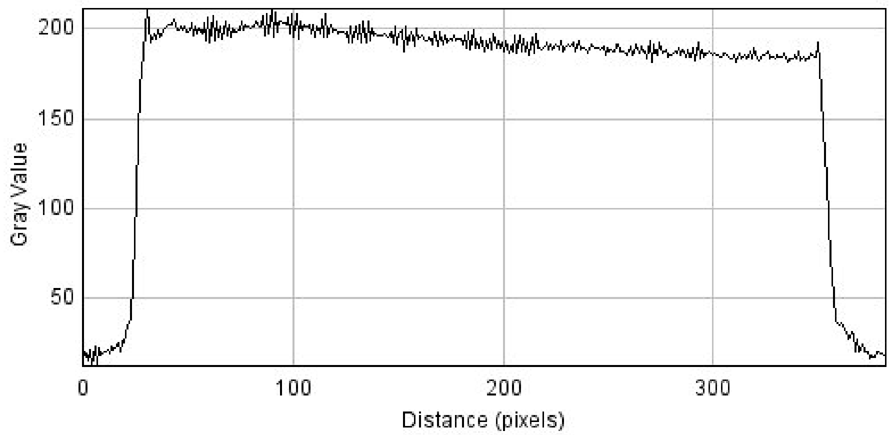

Fig. 4.10 the projection of washer along y axis

The pixel sizes along the x or y axis were then obtained by dividing the real diameter of the ring hole by the pixel distance measured using the profile plots above.

Diameter of hole = 2540µm

We obtained, for the x-profile: pixel distance = 345; for the y-profile: pixel distance = 334

Therefore, on x-axis $\text{Pixel Size} = \frac{2540\mu m}{345} = 7.36\ \mu m$

Similarly, on y-axis $\text{Pixel Size} = \frac{2540\mu\text{m}}{334} = 7.60\ \mu m$

The average pixel size is thus: 7.48 μm

Measuring the gamma factor

Here is the data obtained to measure the gamma factor.

Input beam (I_{in})	**Mean (I_{out})**	**Log I_{in}**	**Log I_{out}**
13.3	68.213	1.123852	1.833867
19	82.273	1.278754	1.915257
28	113.719	1.447158	2.055833
45	156.921	1.653213	2.195681
63	187.704	1.799341	2.273474
88	230.522	1.944483	2.362712

Table 4.10. Values of I_{in} and I_{out} measured

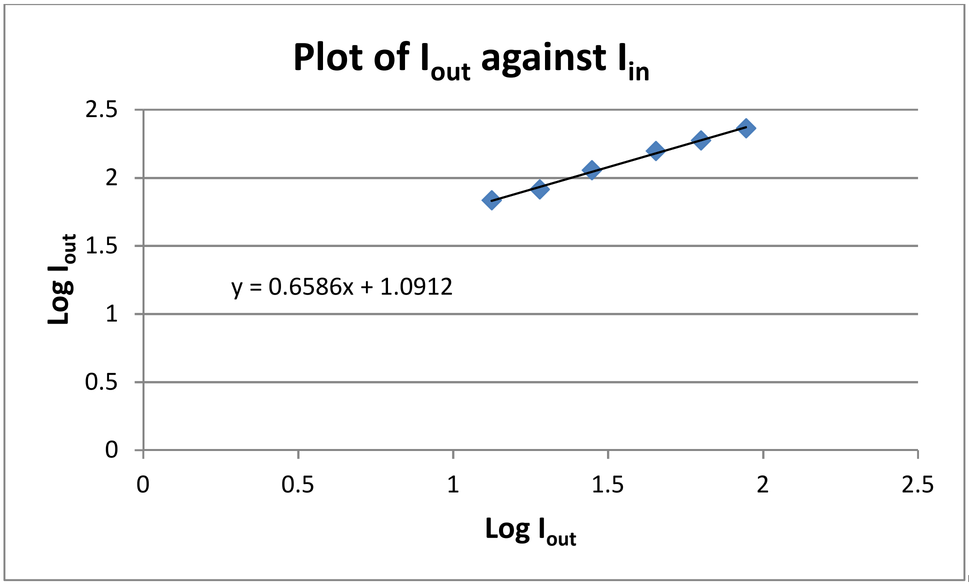

Fig.4.10. Plot of I_{in} against I_{in} showing equation of graph

The gamma factor, from the slope of the graph above, turns out to be: γ = 0.6586

Measuring the laser beam size before the lens

From the Gaussian plots, we obtained,

$$\omega_x = \frac{2(31 \text{ pixel} \times 7.5)\mu\text{m}}{\text{pixel}} = 465\ \mu\text{m}$$

$$\omega_y = \frac{2(35 \text{ pixel} \times 7.5)\mu\text{m}}{\text{pixel}} = 525\ \mu\text{m}$$

Therefore,

$$\omega = \sqrt{\omega_x \omega_y} = \sqrt{465 \times 525} = 494\ \mu\text{m}$$

And by,

$$\omega_{\text{real}} = \omega_{\text{real}}/\sqrt{\gamma}$$

$$\omega_{\text{real}} = 494 \times \sqrt{0.6586} = 400\ \mu\text{m}$$

Measuring the laser beam size of the focused laser beam

An estimation of ω_0 is done using equation (19) from the laboratory manual as follows:

$$\omega_0 = \frac{f\lambda}{\pi\omega_i} = \frac{69000\mu m \times 0.633\mu m}{\pi \times 400\ \mu m} = 34.74\mu m$$

The Rayleigh range is thus,

$$z_R = \frac{\pi{\omega_0}^2}{\lambda} = \frac{\pi \times (35\mu m)^2}{0.633\ \mu m} = 6.08mm$$

The M^2 factor is,

$$M^2 = \frac{\pi{\omega_0}^2}{z_R\lambda} = \frac{\pi \times (35\mu m)^2}{6082 \times 0.633\ \mu m} = 1.00002 \approx 1.0$$

Z(mm)	dx(Pixel)	dy(pixel)
7	4.17	4.34
8	3.8	4
9	3.44	3.59
10	3.27	3.33
11	3.04	3.12
12	2.85	2.96
13	2.65	2.65
14	2.57	2.56
15	2.45	2.44
16	2.61	2.64
17	2.96	2.96
18	3.16	3.28
19	3.33	3.49
20	3.85	3.76
21	3.84	3.91
22	4	4.15

Table 4.10 Results obtained when the laser beam size was measured

As expected, a hyperbolic plot is obtained showing the relationship between the beam radius and the propagation direction, as seen in Fig. 4.10

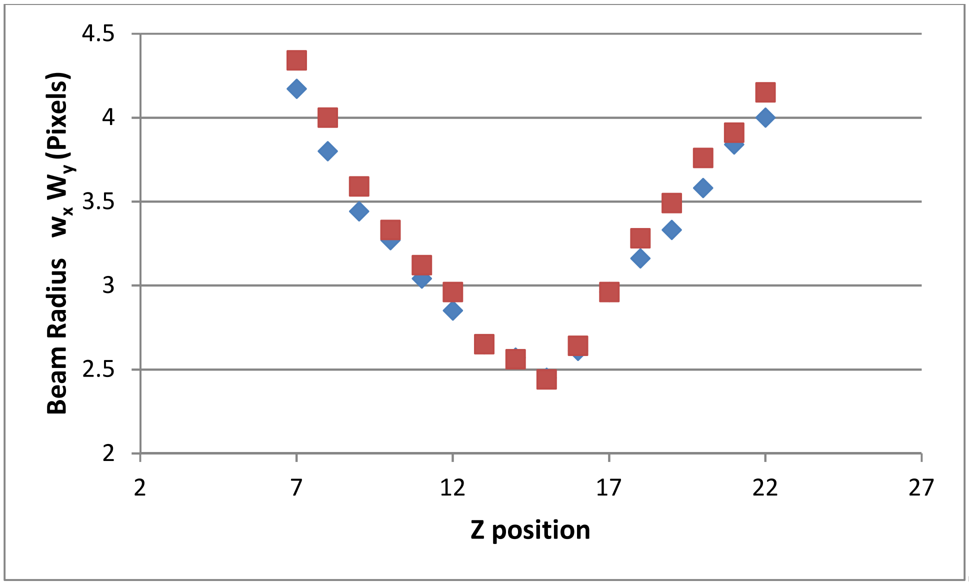

Fig. 4.10. The Gaussian beam radius along the beam propagation direction

5. Conclusions

This experiment showed to be very enlightening as new concepts and methodology were introduced. In the end, we were able to obtain the expected hyperbolic distribution of the laser beam, by plotting the beam radius along the propagation direction, meaning that our methods for conducting the experiment, notwithstanding inaccuracies in observation, are acceptable. We can say that the experiment went well.

In addition, the result obtained for the M^2 factor is very interesting as it shows that the laser source is of high quality.

6. References

[1] Laboratory Manual [PHYS 570 Gaussian Laser Beams by Pengqian Wang (PhD)]

7. Additional questions

7.1 We cannot add the ND filter after the lens because this will shift the focal point and affect the results obtained.

7.2 Derivation of equations

Here we derive equation (24). From equation (21) in the laboratory manual,

$$I_{in} = I_0 \frac{\omega_0{}^2}{\omega_{in}{}^2(z)} e^{-\frac{2(x^2+y^2)}{\omega_{in}{}^2(z)}} \qquad 1.0$$

Let us set $A = I_0 \frac{\omega_0{}^2}{\omega_{in}{}^2(z)}$ so that Eq(1.0) becomes the form,

$$I_{in} = A\, e^{-\frac{2(x^2+y^2)}{\omega_{in}{}^2(z)}} \qquad 2.0$$

I_{out} and I_{in} are thus related by,

$$I_{out} = AI_{in}{}^{\gamma} \qquad \text{, where } \gamma \text{ is the gamma parameter} \qquad 3.0$$

Therefore, we can write,

$$I_{out} = Ae^{(\frac{-2(x^2+y^2)}{\omega_{in}{}^2(z)})^{\gamma}} = Ae^{(\frac{-2(x^2+y^2)}{\omega_{in}{}^2(z)/\gamma})} = Ae^{(\frac{-2(x^2+y^2)}{(\omega_{in}(z)/\sqrt{\gamma})^2})} = Ae^{(\frac{-2(x^2+y^2)}{\omega_0{}^2(z)})}$$

This implies,

$$(\omega_0)^2 = (\omega_{in}/\sqrt{\gamma})^2$$

Or,

$$\omega_{real} = \omega_{real}/\sqrt{\gamma}$$

End of Proof.