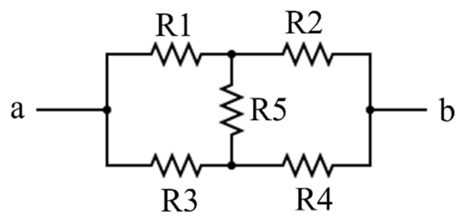
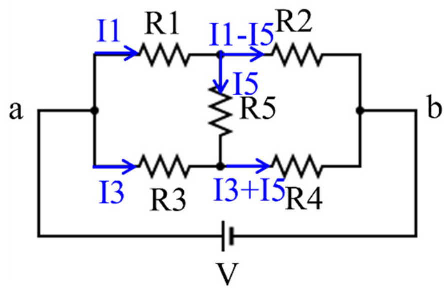
Bridged resistors

Resistor combinations can be more than being in series or in parallel. Here let us find the equivalent resistance for a bridged resistor network, as shown:



In order to use Kirchhoff's laws, we add a power V to the network, and name some currents, as shown:



Applying Kirchhoff's loop laws to the following 3 loops: i) V-R1-R2, ii) V-R3-R4, iii) V-R1-R5-R4, we get

$$V - I_1R_1 - (I_1 - I_5)R_2 = 0$$
$$V - I_3R_3 - (I_3 + I_5)R_4 = 0$$
$$V - I_1R_1 - I_5R_5 - (I_3 + I_5)R_4 = 0$$

Note we have 3 equations and 3 unknowns (I1, I3, and I5). The solution is complicated. Cramer's rule can be used. I screenshot the result from Mathematica:

```
In[1]:= Simplify[
         Solve[V - I1 * R1 - (I1 - I5) * R2 == 0 && V - I3 * R3 - (I3 + I5) * R4 == 0 &&
           V - I1 * R1 - I5 * R5 - (I3 + I5) * R4 == 0, {I1, I3, I5}]]
```

Out[1]=

$$\left\{\left\{I1 \to \frac{(R2\,R3 + R4\,R5 + R3\,(R4 + R5))\,V}{R1\,R2\,(R3 + R4) + R1\,R4\,R5 + R2\,R4\,R5 + R1\,R3\,(R4 + R5) + R2\,R3\,(R4 + R5)},\right.\right.$$

$$I3 \to \frac{(R2\,R5 + R1\,(R2 + R4 + R5))\,V}{R1\,R2\,(R3 + R4) + R1\,R4\,R5 + R2\,R4\,R5 + R1\,R3\,(R4 + R5) + R2\,R3\,(R4 + R5)},$$

$$\left.\left.I5 \to \frac{(R2\,R3 - R1\,R4)\,V}{R1\,R2\,(R3 + R4) + R1\,R4\,R5 + R2\,R4\,R5 + R1\,R3\,(R4 + R5) + R2\,R3\,(R4 + R5)}\right\}\right\}$$

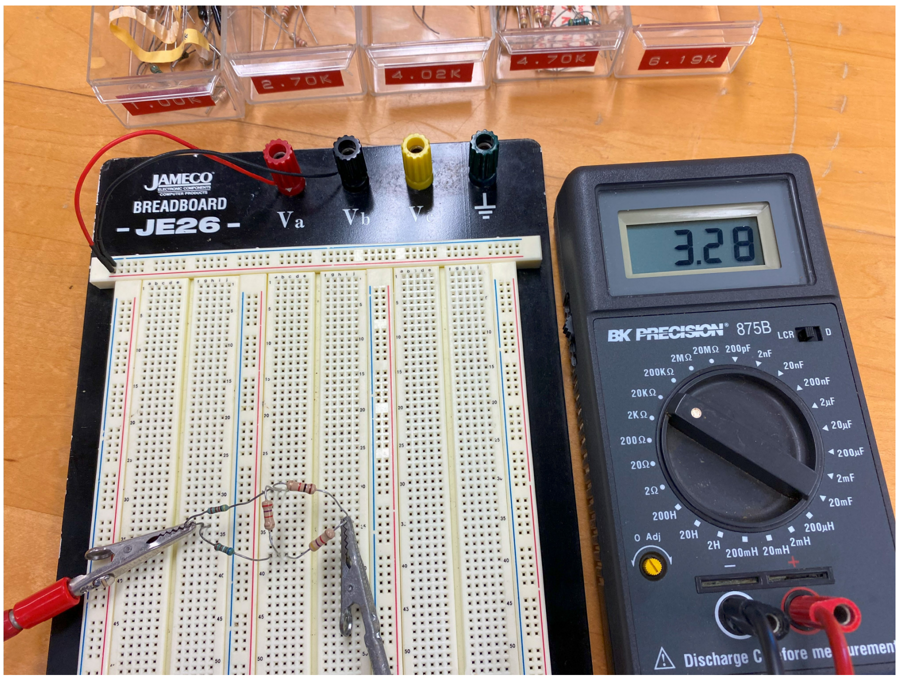
The targeted equivalent resistance is given by R_{ab}=V/(I1+I3), which is

```
In[2]:= FullSimplify[V / (I1 + I3) /. %1]
```

$$\text{Out[2]}= \left\{\frac{R1\ R2\ R3 + R1\ R2\ R4 + R1\ R3\ R4 + R2\ R3\ R4 + (R1 + R2)\ (R3 + R4)\ R5}{(R1 + R3)\ (R2 + R4) + (R1 + R2 + R3 + R4)\ R5}\right\}$$

This is not trivial if we want to do it by hand.

I then went to our WIU physics electronics lab, and used available and arbitrary resistances as R1= 3.98 kΩ, R2= 0.96 kΩ, R3= 6.13 kΩ, R4= 4.62 kΩ, and R5= 2.70 kΩ. Note they are in the same order of magnitude. The measured equivalent resistance is R_{ab}= 3.28 kΩ, as shown:



This matches the calculated value:

```
In[5]:= %2 /. {R1 → 3.98, R2 → 0.96, R3 → 6.13, R4 → 4.62, R5 → 2.70}

Out[5]= {3.28378}
```

Pengqian Wang

September 8, 2025