Physics 214

Modern Physics

Lab Manual

Spring 2024

Department of Physics

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Physics 214 Lab Report Guidelines

Students are required to read the lab manual, and collect their questions before coming to the lab. Each student should have a notebook for the lab. Please bring with you some common stationery to the lab, including a pen, a calculator, a flash drive, and a camera.

Laboratory report for each lab is due in one week following the date of the completion of the experiment, unless otherwise arranged. No absences from the labs or late lab reports can be accepted, except for pre-approved and extenuating circumstances.

All text portions of the lab reports should be prepared using Microsoft Word or a compatible word processor. All graphs in the lab reports should be prepared using Microsoft Excel or a compatible graphing program. Hand-drawing diagrams may be attached to the lab report. If you use a photo, please crop the photo so that only necessary information is shown. You should submit a printed hardcopy for the lab report. A sample student lab report will be posted online.

All lab reports should have the following organizational structure:

1) Title page — Title of the lab experiment, date of the lab experiment, your name, group members' names, course number, and instructor for that lab.
2) Introduction — Explain the purpose of the experiment. Make a brief statement of what the experiment is about. Explain possible applications of the physics you learn in that lab.
3) Background principles — Include and explain any relevant background physics concepts and/or mathematical equations used in data collection and experimental analysis. Include here any derivations asked for in the manual which are necessary for analyzing the data.
4) Experimental procedures — Include a complete description of the instrumentation used, the manner in which you used it, and the quantities to be measured. Sketch the equipment detail and the experimental setup. You may also include cropped photos. The sketches and photos should be supplemented with legible notes. Identify the strengths and weaknesses of the experimental setup, including any special cautions observed. Describe the chronological procedure of how you conduct the experiment, including demos performed by the instructor, problems encountered in the course of the lab, and how you overcame them.
5) Results and discussion — Present your experimental findings, along with relevant data tables, data printouts, graphs, spectra, or images obtained. Describe how you analyze and interpret your data in light of the theory developed in part 3 to reach your final results. Answer questions that are raised in the lab manual.
6) Conclusions — Summarize the major findings and accomplishments of your experiment. Explain any errors incurred or discrepancies with theory, and identify where further work may be applicable.

Physics 214 Computer Simulations: Relativity Paradoxes

The physics of special relativity is the study of very fast but uniform motions. At a high speed that is comparable to the speed of light, velocity, length, time and energy are all dependent on the observer's motion relative to the system he or she is observing! On the other hand, the laws of physics, and the "cause and effect", should not be dependent on who is observing. For example, if a bird dies for one observer, all observers should see that the bird dies. Also if a hunter fires a gun, which then causes the bird to die according to one observer, all observers should agree about the sequence. The most important law that is not relative is the wave equation for electromagnetic radiation in a vacuum. That law sets the speed limit, $c = 3.00 \times 10^8$ m/s, for the universe. All the odd consequences of relativity are in fact a result of the non-relative nature of physics laws in general, and for electromagnetic radiation in particular.

The physics of special relativity have been established through numerous experiments, such as the observed time dilation of relativistic subatomic particles, the Doppler shift of light frequency, and the stunning and destructive fact that $E = mc^2$, which scientists have used for good (nuclear fusion and fission) and bad (atomic weapons). In spite of the numerous tests undertaken to demonstrate the validity of the theory, it is not easy for us to do a hands-on experiment on relativity because of the high velocities required. However, the best way to experiment with relativity is to do it like Einstein did — by thought (German: *Gedanken*) experiment. With a little help from our computers, you will think through some of the most famous paradoxes in special relativity, and hopefully gain an understanding of how velocities, lengths and time change for motions near the speed of light.

You need to learn how to use the program RELLAB in the first lab. They are quite easy for young people. You will become experienced after some practice. Your lab report should follow the general guidelines as described in the beginning of the lab manual. Especially you should include the following:
(a) Sketches of the scenarios. Cropped photos are preferred.
(b) Descriptions of the scenarios.
(c) Your analyses of the paradoxes prior to conducting the simulations. You get the credit no matter your guess is correct or wrong.
(d) Answers to the questions posed in the manual.
(e) The final resolution of the paradoxes.

Good luck and enjoy!

This first exercise should help you develop a feeling for how the radiation of light is different from the radiation of particles or sound. Rather than *answer* questions, your task in this exercise is to *raise* as many questions as you possibly can about what you observe. List everything that strikes you as puzzling, fascinating, or just plain weird! You should have at least ten items on your list. After you write out each of your questions, try to come up with an explanation by experimenting with RELLAB. Don't be too worried if you can't answer all of your questions. Truly insightful questions are the most difficult to answer. If you can't answer a question, write down any and all ideas you have about it, and then try to figure out what in particular is hanging you up.

At several points you will be asked to predict what will happen before you run a scenario on RELLAB and to write down your prediction. This isn't a test. It's just a way to help you start thinking about the problem. Writing out your predictions will allow you to keep track of how your ideas have changed, not only during this one exercise but during your entire study of relativity. As you work through this exercise, note how the RELLAB scenarios compare with your predictions. This should provide a good starting place for developing your list of questions.

Part I: Low Speed Scenario

The first scenario to consider is a bomb explosion at "normal" speeds.

> A robot-piloted airplane flying over the desert is programmed to explode just as it passes above an abandoned car. The explosion causes a loud boom and spews plane and robot fragments in all directions.
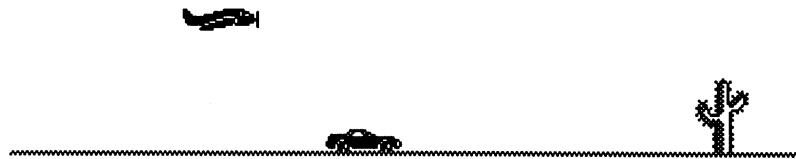
Before building the scenario in RELLAB, write down your predictions for questions 1 and 2 below. You can assume the following factors:

- The speed of sound (330 meters/second = 0.330 kilometers/second) is measured with respect to air, which is at rest relative to the car.

- The speed of the plane's fragments is measured with respect to the plane.
- The plane fragments move in straight lines with constant speed. They are not affected by gravity or air resistance. (This is approximately true for rapidly moving fragments.)

study questions

1. If the fragments are emitted from the explosion at the speed of sound, which will get to the car first: the noise or the fragments? Or will fragments and sound arrive at the car at the same time?
2. In what order do you expect the fragments and sound to reach the cactus further down the road from the car? Which will get to the cactus first?

You will need a working scenario before you can make predictions for questions 3 and 4.

3. After you have run the scenario in the reference frame of the car, make a series of two or three rough sketches showing what you think the scene will look like as observed from the airplane's reference frame.
4. How will your predictions for questions 1 and 2 change when the scenario is run from the airplane's reference frame?

building the scenario

If you prefer to build the scenario on your own, feel free to do so. On the other hand, if you wish to begin with a partially completed scenario, use the file BombSTART on the RELLAB disk. This is a locked file; in order to change the scenario, you must save it as a new file using Save As ... from the File menu.

The following exercise assumes you have used the BombSTART file.

When you begin, the plane is at rest with respect to the car. The first thing you need to do is give the plane a velocity. For starters, set the plane's velocity to 0.2 kilometers/second with respect to the car—you can experiment with different plane velocities after you get the scenario running. To set the plane's velocity,

Double-click the plane and type **0.2** in the velocity box

Check that the velocity is being measured "with respect to car."

Run the scenario by pressing the Run button on the Frame Clock:

▶

You will see that the plane flies over the car.

Stop the scenario by pressing the Stop button:

■

Then, return the clock to zero:

[0.0]

Now create an event in which the plane explodes as it flies over the car. The plane should be directly over the car at time = 00.000. Verify the position of the plane and the car by putting the Cursor/Arrow on each of them and reading the x coordinate at the top of the screen. Both the plane and the car should be located at $x = 0.0$. If the plane is not at position $x = 0.0$,

Double-click the plane to view its dialog box

Type **0** in the slot for x position

When you have made the change,
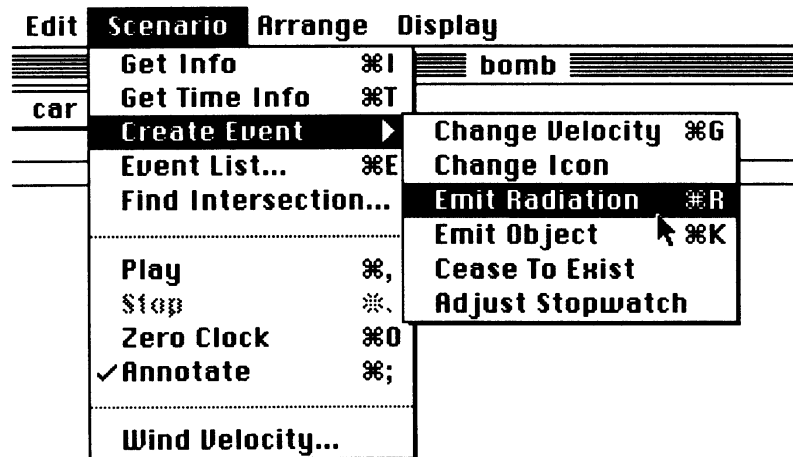
Click the OK button

Now create the explosion event.

Click the plane to select it

A box should appear around the plane. If the dialog box appears instead, click Cancel and try again.

Select Create Event from the Scenario menu

A new menu will pop up to the right.

Keep holding down the mouse button as you drag the cursor onto the new menu and select Emit Radiation...

Edit	**Scenario**	Arrange	Display		
	Get Info	⌘I	≡ bomb ≡	
car	Get Time Info	⌘T		
	Create Event	▶	Change Velocity	⌘G
	Event List...	⌘E	Change Icon	
	Find Intersection...		**Emit Radiation**	⌘R
			Emit Object	⌘K
	Play	⌘,	Cease To Exist	
	Stop	⌘.	Adjust Stopwatch	
	Zero Clock	⌘0		
	✓Annotate	⌘;		
	Wind Velocity...			

Note that if the Create Event option is "grayed out," it means that no object, or more than one object, has been selected. Go back and re-select the plane by clicking it again.

When the dialog box appears,

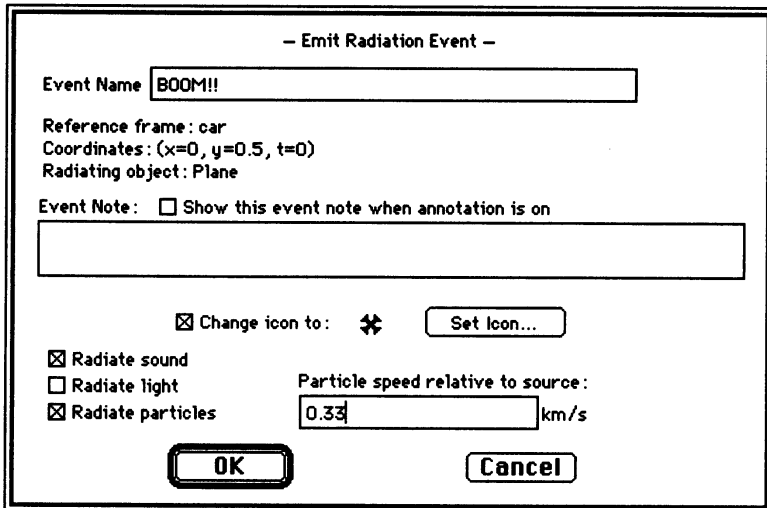Type in a name for your event, for instance, **BOOM**

When the plane explodes, you will want its icon to change (since it's no longer going to be a plane).

Click the small Change icon to ... box

Then click the Set Icon... box and choose a new icon for the remnant of the plane

Click both the Radiate sound and Radiate particles boxes, and type **0.33** (kilometers/second) for the particle speed relative to source

The completed dialog box for the event should look something like this:

```
┌──────────────────────────────────────────────────────────┐
│                    — Emit Radiation Event —                │
│                                                            │
│  Event Name │BOOM!!                                    │   │
│                                                            │
│  Reference frame: car                                      │
│  Coordinates: (x=0, y=0.5, t=0)                            │
│  Radiating object: Plane                                   │
│                                                            │
│  Event Note:  ☐ Show this event note when annotation is on │
│  ┌──────────────────────────────────────────────────────┐ │
│  │                                                      │ │
│  └──────────────────────────────────────────────────────┘ │
│                                                            │
│               ☒ Change icon to:  ✷    [ Set Icon... ]      │
│  ☒ Radiate sound                                           │
│  ☐ Radiate light        Particle speed relative to source: │
│  ☒ Radiate particles    │0.33              │ km/s          │
│                  ( OK )              ( Cancel )             │
└──────────────────────────────────────────────────────────┘
```

Notice that at time zero the explosion event has just occurred. In order to view the scenario prior to the explosion, you have to start the clock at a negative time.

To do this,

Click the Cursor/Arrow on the clock digit just to the left of the decimal place (whole seconds)

Arrows should appear above and below the digit.

```
┌═══ Clock ═══┐          ┌═══ Clock ═══┐
│ sec ▼ + 00.000│  »» "CLICK" »»  │ sec ▼ + 00.000│
│ ▶ ■ 0.0 ◁◁▷▷│          │ ▶ ■ 0.0 ◁◁▷▷│
└─────────────┘          └─────────────┘
```

Click the down arrowhead several times and watch the plane

Do this until you set the clock to –2 or –3 seconds. (You can change any of the clock digits in this way.) Your scenario is now ready to go.

Click the Run button

▶

and you're off!

As you compile your list of ten or more questions, try to answer the four questions listed at the beginning of this scenario. Then experiment. Do the answers change for a faster plane? Do they change for a slower plane?

Note that to change the plane's velocity, first zero the clock, then double-click the plane to view its dialog box. Type the new velocity and click OK.

Save your scenario from this part of the exercise before going on to the next part.

Part II: High-Speed Scenario

The next scenario to consider is a bomb exploding in space, emitting light and fragments.

> An extremely powerful bomb, floating in space, explodes when a passing rocket touches it. The explosion creates a brilliant flash of light and ejects bomb fragments in all directions at about half the speed of light. Fortunately for the rocket's crew, the protection shields are activated and the ship is barely damaged.

> Assume the rocket travels at a significant fraction of the speed of light (> 0.5 light-seconds/second) with respect to the bomb. Read through the following questions, and then build a RELLAB scenario to help work out your answers (see the following section on building a scenario).
>
> 5. What is the width of the screen in your new scenario? Roughly how many times bigger is it now than the last scenario?
> 6. After building the scenario and running it from the bomb's reference frame, make a series of two or three rough sketches showing what you expect it to look like from the rocket's reference frame.
> 7. How do the radiation of light and the "radiation" of particles differ?

building the scenario   This is a good scenario to practice building from scratch.

> Select New... from the File menu
>
> Name (and save) your new scenario by selecting Save from the File menu and completing the dialog box
>
> Now you are ready to start building your scenario. First change the scale of the scenario window. Recall that light travels 300,000 kilometers in a second, so in order to keep track of light and objects traveling at nearly the speed of light, you need a scenario window several millions of

kilometers across. To make the numbers less formidable, choose a more convenient unit of length: the light-second. One light-second is just the distance traveled by a flash of light in one second, or 300,000 kilometers.

You can reset the length units and reset the width of the scenario window in one quick step, as follows:

First, find the arrow that shows the width of the scenario window

←————————————————— 53.9 [**meter**] ————————————————→

While holding down the Option key on your keyboard, move the Cursor/Arrow to the box that says meter, and click and hold the mouse button (the Option key is near the lower left of the keyboard)

The following menu appears:

≡ **rocketbomb** ≡
	mm
	cm
53.9	✓meter
	km
	light sec
	light min
	light hr
	light day
	light yr
	parsec
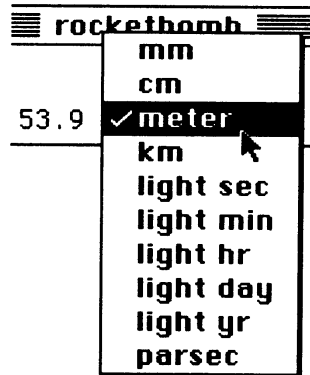
Move the Cursor/Arrow to light sec and release the mouse button

The new width arrow should look something like this:
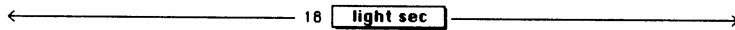
←————————————————— 18 [**light sec**] ————————————————→

Note that if, instead, you see something like this:

←————————————————— 1.8E-07 [**light sec**] ————————————————→

it means that RELLAB changed the length units from meters to light-seconds but that it didn't make the window bigger. (When you select a new unit of length from the menu without holding down the Option key, the computer doesn't change the size of the window.) You will need to change the length units back to meters, this time without holding down the Option key, and then repeat the process above.

To fine-tune the size of the window,

Click the Zoom In tool

⊠

or the Zoom Out tool

With the Zoom In tool, you draw a box on the screen by clicking and dragging with the tool selected. When you let go of the mouse button, everything inside that box expands to fill the window. With the Zoom Out tool, you draw a box and everything in the window shrinks into the box. With either tool, clicking the mouse without drawing a box will make the screen expand or contract by ten times around the position of the cursor arrow.

Now that your scenario window is big enough to handle objects moving near light-speed, create some of these objects.

Change the original object (the dot) to the bomb by double-clicking the dot,

Select Set Icon..., and then double-click the bomb icon

Type the object's name in the remaining dialog box and click OK

You want the bomb to explode at time zero, so check to see that the clock is zeroed.

Select the bomb (box appears around it), and then select Create Event... Emit Radiation from the Scenario menu

Complete the dialog box to look like the following screen and then click OK

— Emit Radiation Event —

Event Name | BANG!!

Reference frame : Object 1
Coordinates : (x=0, y=0, t=0)
Radiating object : Object 1

Event Note : ☐ Show this event note when annotation is on

☒ Change icon to :   ✸   [Set Icon...]

☐ Radiate sound
☒ Radiate light   Particle speed relative to source :
☒ Radiate particles | 0.5 | | lt-s/s

[OK]   [Cancel]

Back the clock up a few seconds (see the instructions for building and running the low-speed scenario), and run the scenario to check that everything is OK

Now add the hapless rocket. To do this,

Reset the clock to zero (since you want the bomb to explode just as the rocket touches it)

Select the Create Object tool

☐•

Place the cross hairs directly over the bomb (or the bomb remnant, if its icon changes during the explosion), and click

Choose a right-facing rocket and click OK

In the next dialog box that appears, check that the position of the rocket is $x = 0$, $y = 0$. If not,

Type 0 in each of the boxes for the x and y positions

Type in a speed between 0.6 and 0.8 light-seconds/ second for the rocket with respect to the bomb

The completed dialog box should look something like the screen below.

Click OK

```
┌──────────────────────────────────────────────────────────────────┐
│  Name │Rocket                                    │  ⊏⊐►  (  Set Icon... )│
│                                                                    │
│  Frame : Bomb                                                      │
│  ☐ Show time      ☐ Show velocity      ☐ Show name                 │
│                      x                          y                  │
│  Position*  │0              │lt-s    │0              │lt-s          │
│                   speed            •        direction              │
│  Velocity *... │0.7          │lt-s/s  │0              │°            │
│     ...with respect to : [  Bomb  ]                                │
│  Time stretch factor : 1.40028008403                              │
│    ( ▇OK▇ )    ( Cancel )        ☐ Change group members to match   │
└──────────────────────────────────────────────────────────────────┘
```

Your new scenario is ready for a test flight. Back up the time a few seconds and click the Run button on the clock:

▶

Explore this scenario and develop questions of your own. Recall question 6 before viewing the scenario from the rocket's reference frame. Also try questions 5 and 7 as you work.

End of Exercise

In the late 1940's one of the great challenges in aviation was the quest to fly a plane faster than the speed of sound. Nobody, scientist nor pilot, knew what would happen to an airplane if it "broke the sound barrier." Nobody knew if this feat was even possible. Design after design produced faster and faster planes, breaking speed record after speed record, inching ever-closer to the mystical speed of sound. Pilots approaching the speed of sound were said to be "pushing the envelope"; pushing their planes and themselves to the very edge of what was possible. In 1947, Chuck Yeager flew the Bell X-1 through the sound barrier and lived to tell about it. Today, supersonic airplanes are almost commonplace.

Can we "push the envelope" with light? Can we possibly punch through the "light barrier"?

Special relativity tells us that nothing can move faster than the speed of light in a vacuum. Maybe so, but why can't a clever choice of reference frames allow us to observe something going faster than the speed of light? We know, for example, that if we cruise down the highway at 55 mph and a car zips along toward us in the other lane at 55 mph, a radar gun mounted on our car measures the speed of the other car to be 110 mph. What happens if one rocket going 0.8 light-second/second with respect to the Earth passes another rocket going 0.8 light-second/second with respect to the Earth but moving in the opposite direction?
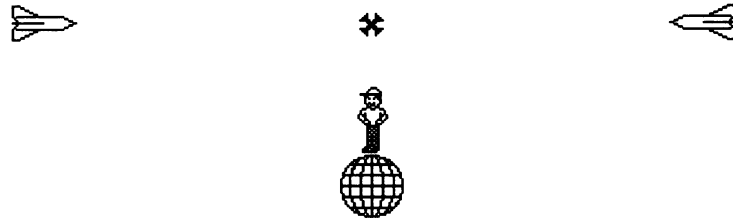
Can you think of other possible ways of beating the speed of light? Think of as many ways to break this barrier as you can, jot them down in words and pictures, and then test them out using RELLAB. Keep track of your observations, questions, and discoveries. Discuss them with others. When you are finished, turn in your written conclusions.

If you don't feel comfortable building your own scenario from scratch, or if you don't have time, use the "starter kit" called EnvelopeSTART. Again, this is a locked file. You need to call it up and save it under another name in order to change it.

Taking It Further

Set the speed of the left-hand rocket to 0.8 the speed of light with respect to the planet (rocket moving to the right, direction: zero degrees) and the right-hand rocket moving to the left (direction: 180 degrees) at 0.8 the speed of light. Then it would seem that one rocket measures the speed of the other rocket as 0.8 + 0.8 = 1.6 times the speed of light. Does RELLAB agree? Check this by jumping to the frame of one rocket (menu in upper left corner of the screen) and double-clicking the other rocket to examine its speed in the frame of the first rocket, as it appears in the dialog box.

Experiment with a variety of equal-but-opposite velocities as observed in the Earth frame. Then plot on the following graph the rocket velocity with respect to the Earth bystander (horizontal axis) versus the rocket velocity with respect to the other rocket (vertical axis). Above what speed do you start to notice that the velocities do not add in the normal way?

Explain in a paragraph what the shape of your graph means.

v in light-seconds/second
(rocket with respect to other rocket)

v in light-seconds/second
(rocket with respect to Earth)
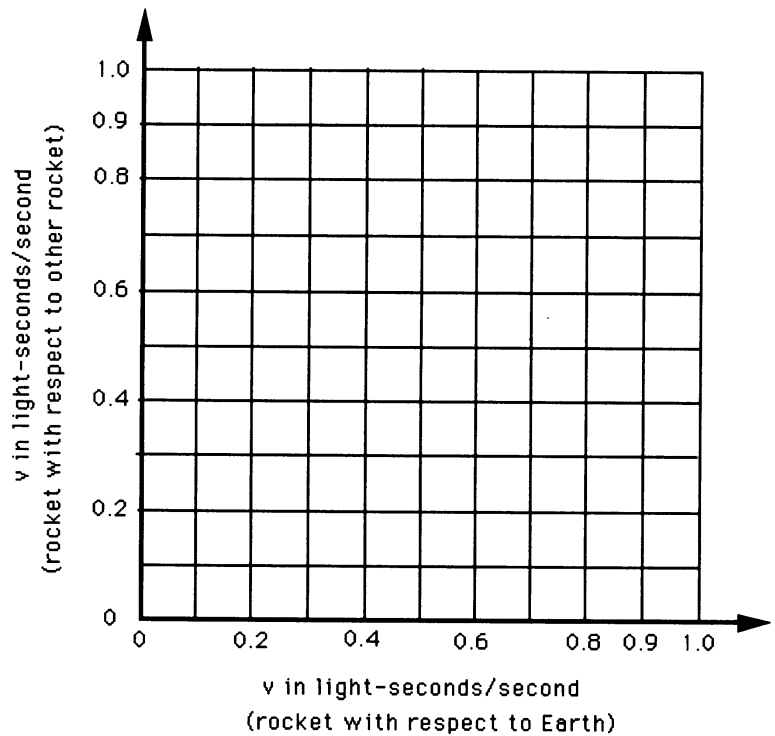
End of Exercise

Zeena   Burger Palace   Earth

Your friend Zeena from planet Zxorq decides to pay you a visit on Earth. She hops in her space saucer and sets out for Earth at 0.8 the speed of light. As she passes the Intergalactic Burger Palace at full speed, exactly halfway between Zxorq and Earth, she sends two radio messages (at the speed of light): one to you, letting you know she's on her way, and one to her Mom, saying that she'll be late for dinner. Recalling the physics she just learned that week in school, Zeena figures, "The message I sent to Earth will arrive well before the message I sent to my Mom since I'm moving toward Earth and away from my Mom."

1. Is your friend confused about her physics? Do you agree with her prediction? Rewrite her statement to avoid misunderstandings and to conform to your prediction.
2. Make two rough sketches showing where Zeena expects the planets to be, relative to her rocket, when each of her messages arrives at its destination.
3. No one is home at your house or at Zeena's house when her messages arrive, so they are picked up by answering machines that automatically record the time of each message. If the clocks on Earth and on Zxorq are synchronized with one another, in what order do the messages arrive, according to the answering machines? (Later, while visiting you, Zeena writes down the time recorded on your answering machine; then, when she gets home, she compares it to the time recorded on her Mom's answering machine.)
4. Make a rough sketch showing where a person standing on Earth would expect Zeena's rocket and the other message to be when her message arrives at Earth. Make another sketch indicating where a person standing on Zxorq would expect Zeena to be when her message arrives at Zxorq.
5. List three things that you find puzzling or confusing about this scenario.

After you have answered these questions, build the scenario on RELLAB. You can build it from scratch or use the pre-made RELLAB "starter kit" named FriendSTART. To avoid confusion, do not have Zeena's spaceship stop at Earth—just let it fly on past. When you get your scenario running, use it to check your answers to the questions and to explore things you find puzzling. Don't change your original predictions; instead, make notes about what you learn as you go. Report your final conclusions and also how and why your reasoning changed between initial predictions and final answers.

End of Exercise

The President of Galactic Region Twelve worries that he is losing popular support. He decides to fly past all of the planets in the Region to show how deeply he cares about each and every citizen. On his trip, the President rides in a luxury space cruiser protected by two Hyper-Falcon 16 attack ships. The Hyper-Falcons fly in formation at the same speed as the President, with his cruiser exactly halfway between them. The President and his escorts travel at a significant fraction of light speed (> 0.5 c) with respect to Earth, as shown below:
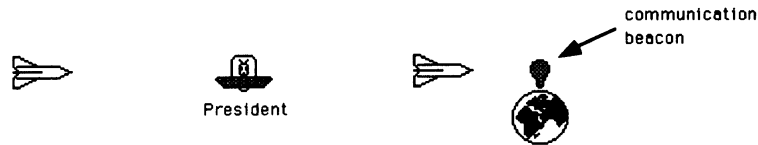
As the entourage passes Earth, an Earthbound campaign aide thinks she notices that the President's name is misspelled on his re-election banner. In order to avoid undue embarrassment, the aide sends a message to the two escort ships asking them to check the banner and, if necessary, to fix it without letting the President know. The message travels at the speed of light. The President's cruiser is at exactly the same location as the communication beacon when the aide sends her message, as shown below:

Beacon flashes message just as President passes.

1. Does the aide expect the right or the left escort to receive her message first?
2. The President sees the flash of the communication beacon and suspects something is up. Does the President expect the right escort or the left escort to receive the message first?
3. Physics should work the same way for everyone. Does everyone agree about which escort receives the message first?
4. As soon as each Hyper-Falcon receives the message, it speeds toward the President at 0.9 light-second/second (with respect to the President). Which escort reaches the President first? Does the aide on Earth agree with the President about who arrives first?

Set up a RELLAB scenario to answer these questions. Start from scratch or use the starter file PresidentSTART.

Taking It Further

Once you have a working scenario and have answered the previous questions, put clocks on the President's cruiser and on Earth. To do this,

Click the object whose clock you want to see

A box should appear around the object indicating that it has been selected.

Select the Show Object Time command from the Display menu

If this option is "grayed out" it means that no object is selected and you must repeat the preceding step.

A small box containing numbers will appear below the object, connected to it by a line. The clock can be moved to a new position by clicking it and dragging it with the Cursor/Arrow.

It will be easier to compare the readings on the clocks if you advance the time digit by digit on the main Frame Clock, rather than running the scenario automatically.

Record the Earth time and the President's time for the events "Left Escort Reaches President" and "Right Escort Reaches President." (It may be helpful to annotate these events and toggle "Annotate" on under the Scenario menu. That way the scenario will stop running and give you time to record your data.) Make two sets of measurements, one while running the scenario in the Earth frame, and the other while running the scenario in the President's frame.

Explain the differences between measurements made in the two frames. At what time does the left escort *really* reach the President's cruiser? (Look out for the word *really* in relativity!)

End of Exercise

A letter sent to the Massachusetts Institute of Technology by Hsien-Yen Tsao of Los Angeles poses the following paradox, which he asserts disproves the theory of relativity. The Chairman of the Physics Department sends the inquiry to you, asking you to respond to Mr. Tsao. You are determined to make the answer clear, concise, decisive, and polite—a personal test of both your diplomacy and grasp of relativity.

The Setting

A train travels at high speed. A runner on the train sprints toward the back of the train with the same speed (with respect to the train) as the train moves forward (with respect to Earth). Therefore, the runner is not moving with respect to Earth.

The Paradox

You know that, crudely speaking, "clocks on the train run slow" compared to the Earth clock. You also know that "the runner's clock runs slow" compared to the train clocks. Therefore, the runner's clock should run "doubly slow" with respect to the Earth clock. But the runner is not moving with respect to Earth! Therefore, the runner's clock must run at the same rate as the Earth clock. How can it possibly be that the runner's clock runs "doubly slow" with respect to the Earth clock and also runs at the same rate as the Earth clock?

Here you let the runner ride a bicycle toward the rear of the train so that the "runner" is at rest with respect to a cactus growing on Earth.
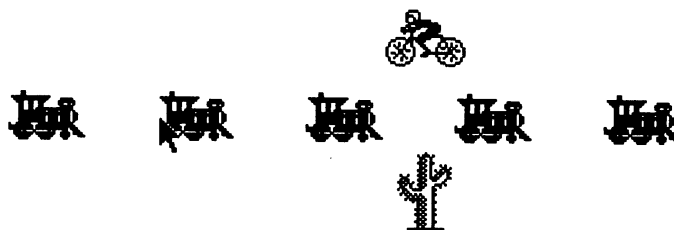
Build a scenario in RELLAB similar to the one shown in the figure or use the starter file called TrainRunnerSTART. If you are building your own scenario, read the following suggestions:

1. Build the train at rest, then add the runner and cactus moving to the left. Then jump to the cactus frame.

This will simplify the job of setting the velocity of all train cars.

2. At time zero, the first train car, the runner, and the ground icon should all line up vertically. This way their clocks will all read "0.000" at the beginning of the scenario.

3. Leave large spaces between the train cars (you will notice that they get closer together when you switch to the ground reference frame).

The runner and the cactus should have the same velocity (both speed and direction) with respect to the train. (Switch between frames and check the velocities of the train and runner to convince yourself that they are consistent with Mr. Tsao's description of the scenario.)

Hints

You want to compare the rate at which the clocks run in each of the reference frames. Once you have the basic scenario built and running, you need to add clocks to the objects. To do this,

Click the object whose clock you want to see

A box should appear around the object, indicating that it has been selected.

Select Show Object Time from the Display menu

If this option is "grayed out," it means that no object is selected and you need to repeat the preceding step.

A small box with numbers in it should appear below the object, connected by a line. The clock can be moved to a new position by clicking and dragging using the Cursor/Arrow.

It will be easier to compare the readings on the clocks if you advance the time digit by digit on the main scenario clock, rather than by running the scenario automatically.

Write your letter to Mr. Tsao in one page or less.

Taking It Further

Explain why clocks on different train cars read different times when viewed in the ground frame. Is there any situation in which train clocks agree with one another as observed in the ground frame?

End of Exercise

The Pole and Barn Paradox challenges your understanding of two central results of special relativity: the Lorentz contraction and the relativity of simultaneity. Once you finish the analysis, you will be suspicious the next time someone uses the word *when*.

A worried student writes, "Relativity must be *wrong*. Consider a 20-meter pole carried so fast in the direction of its length that it appears to be only 10 meters long in the laboratory frame of reference. Let the runner who carries the pole enter a barn 10 meters long. At some instant, the farmer can close the front and back doors and the pole will be entirely enclosed in the barn. However, look at the same situation from the frame of reference of the runner. To him the barn appears to be contracted to half its length. How can a 20-meter pole possibly fit into a 5-meter barn? Does not this unbelievable conclusion prove that relativity contains somewhere a fundamental logical inconsistency?"

Construct the RELLAB scenario yourself or use the starter file BarnSTART and save it under another name so you can change it.

The lengths of the pole and the barn have been changed from meters to light-seconds so you can play the resulting program in real time—a few seconds.

You see the "barn" in the middle of the screen with doors closed at $t = 0$.

> Click time forward and backward to show that the doors open for both positive and negative times

> Reset time to zero and build a moving pole inside the barn, the same length as the barn, using right and left arrowheads moving at speed 0.866 light-second/second

Use the line tool to connect these two arrowheads. If you wish, put a runner in the middle of the pole.

> Click the clock back to negative time so the pole is clear of the barn and run the scenario in the barn frame

> Now jump to the pole frame (Frame menu at upper left of the screen) and run the scenario again

Draft the beginning of your reply to the worried student by answering her question, "How can a 20-meter pole possibly fit into a 10-meter barn?"

Is there some way to squeeze the paradox harder? What happens if the pole stops at the instant it is entirely contained in the barn? There are at least two ways to stop the pole: (1) stop all parts of the pole at the same time, or (2) let

the pole run into the back end of the barn. Instructions are given for method (1), leaving you to explore method (2) with some hints.

Method 1

Stop all parts of the pole at the same time. Replace the pole with, say, five tennis balls along a line the same length as the pole and moving together with the same velocity as the pole. The farmer's five children line up inside the barn, and each child catches and stops one tennis ball at the same time (in the barn frame) as the farmer closes the front and back doors of the barn. Call up BarnSTART again and rename it as **BarnTennis** or some such name. Zero the clock and place five tennis balls inside the barn. Give each tennis ball a velocity of 0.866 with respect to the barn. At $t = 0$ (the time the clock now reads), you want all the tennis balls to come to rest in the barn frame. You will have to create a "change velocity" event for each tennis ball. After doing so, back the clock up and run the scenario in the barn frame to check that the tennis balls all stop at $t = 0$.

Predict the sequence of events recorded by an observer riding on the last (left-hand) tennis ball. Now jump to the frame of the left-hand tennis ball, click the time backward until the tennis balls are clear of the barn, and run the scenario. At the end, the barn changes length suddenly. What is going on?

Method 2

Let the pole run into the back end of the barn. Suppose the barn has no back door but, rather, has a back wall of steel-reinforced concrete. What happens after the farmer closes the front door on the back end of the pole?

Hints

Forces are ordinarily transmitted through an object at the speed of sound, and sometimes at the speed of a shock wave. The fastest a signal or "influence" can move is at the speed of light with respect to any given inertial frame. How quickly is it *possible* for the back of the pole to get a message that the front of the pole has stopped? What if you emit a light flash from the event at which the front of the pole reaches the back of the barn? Does this flash arrive at the back of the pole soon enough to keep it from entering the barn?

Finish your answer to the worried student.

End of Exercise

A U-shaped structure made of the strongest steel contains a detonator switch connected by wire to one metric ton (1000 kilograms) of TNT, as shown below. A T-shaped structure made of the same strong steel fits inside the U, and the long arm of the T does not quite reach the detonator switch when both structures are at rest.

Both at rest
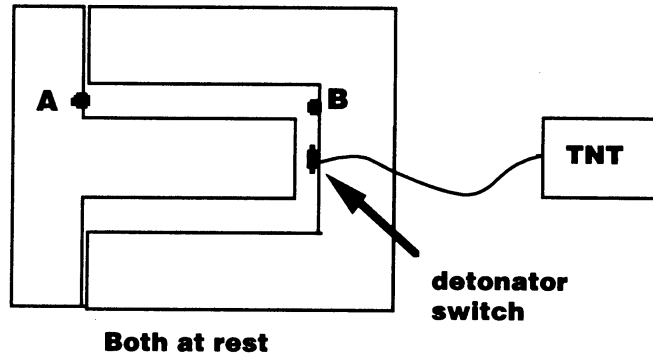
Now the T-structure is removed far to the left and accelerated to high speed. It is Lorentz-contracted along its direction of motion. As a result, the long arm is not quite long enough to reach the detonator switch when the two collide, as shown below. Therefore, there will be no explosion.

Rest frame of U-structure
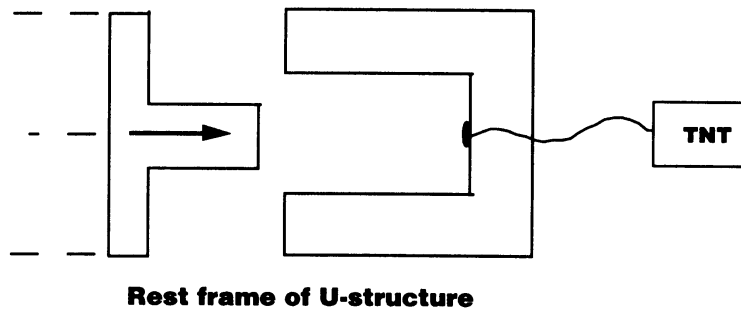
However, look at the same situation in the rest frame of the T-structure. In this frame, the arm of the T has its rest length, while the two arms of the U-structure are Lorentz-contracted. Therefore, the arm of the T will certainly strike the detonator switch, as shown below. There will be a terrible explosion.

Rest frame of T-structure
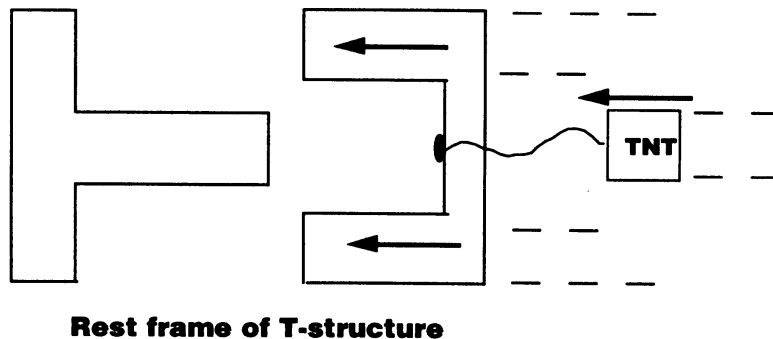
(a) Make a decisive prediction: Will there be an explosion or not? Your life depends on it!

(b) Set up a RELLAB scenario to test your prediction.

(c) The wire from the detonator switch to the TNT is restrung through point B in the first figure, and a laser is installed at point A on the T structure. When the two structures collide at point A, the laser fires a pulse toward point B that severs the wire. Does this new apparatus change your prediction about the detonation of the TNT? After making a prediction, try it out with RELLAB.

End of Exercise

Sir Galahad rushes forward at a full gallop, his sturdy lance aimed dead-center for Lancelot's breastplate. As the knights thunder toward one another, Galahad tightens his grip and braces for the impact, but before he feels his lance connect, a thunderous blow wrenches him from his horse and sends him sailing to the ground. The baffled Galahad drags himself to his feet and stumbles over to inspect his fallen lance. He sees that the front half was neatly sawed off before the joust. He throws down his lance and curses Lancelot. Grinning, the other knight shouts back, "Better luck next time, old friend," at which the crowd roars with laughter.

Shortening your opponent's lance in a jousting match certainly gives you an unfair advantage. Does relativity give you a similar advantage?

Picture this futuristic tournament: Two knights charge toward each other, each traveling at 0.7 the speed of light with respect to the onlooking crowd. Our futuristic Lancelot grins smugly to himself as he approaches center court. This time he checked out both lances by laying them side-by-side to prove they are of exactly equal length when at rest with respect to one another. However, he also knows that special relativity renders the lance of his onrushing opponent shorter than his own as observed in his frame. So on he rides, fully confident of his victory. But our futuristic Galahad, thundering toward Lancelot from the opposite direction, also knows about Einstein's physics, and fully expects to claim an easy victory in this mismatched duel. Who's fooling whom?
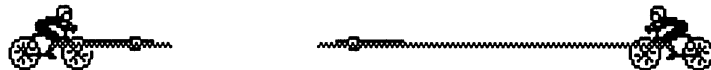
In Lancelot's frame...

Lancelot
(at rest)

Galahad
(moving rapidly to the left)

But in Galahad's frame...

Lancelot
(moving rapidly to the right)

Galahad
(at rest)

Everyone has to agree on the outcome of the joust, and it can only be *one* of the following circumstances:
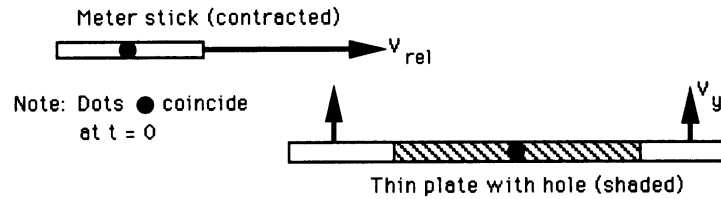
(a) Galahad gets knocked down;

(b) Lancelot gets knocked down; or

(c) Both knights get knocked down.

Before using RELLAB, write down which outcome you believe is correct and explain (clearly and convincingly) how Lancelot and Galahad can both agree on the same correct conclusion.

This is a particularly difficult paradox. Even setting up the scenario in RELLAB may not give you the full answer. Use all the skills and tricks you have learned from the Pole and Barn and Detonator paradoxes—as well as from all your classmates. In your report on this paradox, explain the tests you tried, the process you followed, and your conclusions.

Note that when you build a jousting scenario, the length of the lances must be the same *proper* length, the length of each as measured in *its own rest frame*. If Lancelot measures his own lance to be 2 light-seconds long in his own reference frame, then Galahad should measure his own lance to be 2 light-seconds long in *his* own reference frame.

End of Exercise

Meter stick (contracted)

v_{rel}

Note: Dots ● coincide
at t = 0

v_y

Thin plate with hole (shaded)

LABORATORY FRAME

A meter stick slides along the x axis of the laboratory frame and approaches the origin with velocity v_{rel}. A very thin plate parallel to the x-z laboratory plane moves upward in the y direction with speed v_y. The plate has a circular hole with a 1-meter diameter centered on the y axis. The center of the meter stick arrives at the laboratory origin at the same time in the laboratory frame as the rising plate arrives at the plane $y = 0$. Since the meter stick is Lorentz-contracted in the laboratory frame, it easily passes through the hole in the rising plate. Therefore, there will be no collision between the meter stick and the plate as each continues its motion.

However, someone who objects to this conclusion can make the following argument: "In the meter stick frame (in which the meter stick is at rest), the meter stick is not contracted, while in the lab frame the hole in the plate is Lorentz-contracted. Hence, the full-length meter stick cannot possibly pass through the contracted hole in the plate. Therefore, a collision must occur between the meter stick and the plate."

Choose sides in this argument. Write down your prediction, answering unequivocally the question, "Will there be a collision between the meter stick and the plate or not?"

Set up this scenario on RELLAB. Let the center of the meter stick and the center of the hole coincide at $t = 0$. Play the scenario from the "laboratory frame." Next, jump to the meter stick frame and run the scenario again.

Was your prediction correct? Explain what is happening and why.

End of Exercise

Photoelectric Effect and Planck's Constant

In the early days of radio the transmitter consisted of a simple electric spark which produced radio waves. The radio waves were detected by a coil of wire placed some distance away from the spark. By turning the spark on and off, "wireless" radio communications using the Morse telegraph code were made possible. The photoelectric effect (Fig. 1) was discovered when physicist Heinrich Hertz, who was working to improve radio broadcasting, discovered that the air between the electrodes of a spark gap became a better conductor when lit by ultraviolet light from an arc lamp. In 1888 it was discovered that shining an ultraviolet light on a negatively charged zinc surface caused the zinc to lose its charge quickly. A neutral zinc surface became positively charged under the light. If the zinc was initially positively charged, the charge was unchanged by light. Somehow light was affecting the electrostatic charge of the zinc.

Today we understand the photoelectric effect as the ejection of electrons from a surface due to light striking the surface. The main features of the photoelectric effect are:

1. The quantity of ejected electrons (photocurrent) depends on the light intensity.

2. The light intensity does not affect the kinetic energy of the electrons.

3. No electrons are ejected below a certain frequency (energy) of light.

4. The kinetic energy of the electrons has a maximum value.

5. The kinetic energy of the electrons depends on the frequency (energy) of the light.

Each "investigation" in this lab is a separate mini-experiment designed to test one of these statements. The last statement is particularly important, because it leads to Planck's constant. This constant is one of the fundamental constants of the universe. The last thing that you will do in lab today is to measure Planck's constant.

Use the scientific method when you work. Observe what happens while you change one and only one thing at a time, then draw your own conclusions based on your data. Think about what happened. Then write down your thoughts in your lab book.

The apparatus:

The photocell (Fig. 2) contains a clear window, a photocathode, and an anode. The photocathode is the surface from which electrons (the photoelectrons) are emitted. The ejected electrons flow into the anode and form an electric current (the photocurrent) which is measured on the ammeter. Applying a reverse bias or stopping voltage (charging the anode negative and the cathode positive) prevents the electrons from reaching the anode. The maximum kinetic energy of the electrons in eV is found by multiplying the stopping voltage by the electron charge.
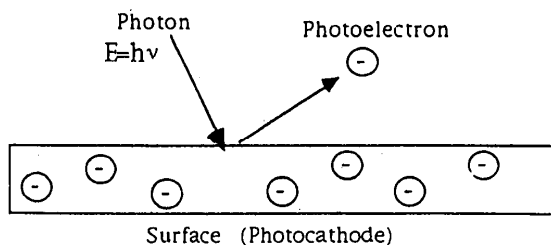
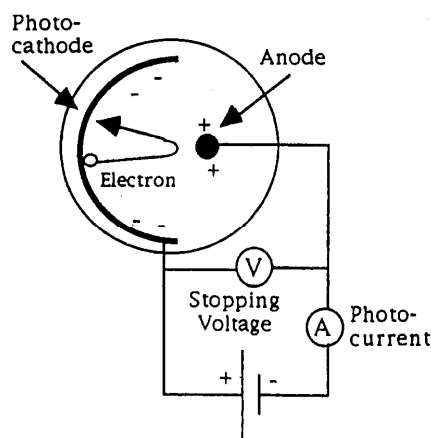Fig. 1: Ejection of a photoelectron   Fig. 2: General photocell setup

Laboratory setup:

Your apparatus consists of a box (Sargent-Welch Cat. No. 2120) containing the phototube and a voltage divider circuit used for controlling the voltages on the anode and cathode. You will be given two photocell boxes. They are identical except that one contains a color filter wheel, and the other contains an aperture wheel. ("Aperture" is the physics talk for "small hole".) The circuit diagram of the box is printed on its front panel. The diagram shows how to connect the phototube to the power supply and meters.

For measuring the photocurrent, we recently bought some sensitive digital multimeters (Tektronix DMM 4020), which have a current resolution of 1 nA, a voltage resolution of 10 μV, and a resistance resolution of 1 mΩ. These multimeters have replaced the large antique galvanometers, which started to function improperly in the last few years and have been removed from our lab. When using the digital multimeters, please keep in mind that for measuring very small current (a few nanoamperes), the multimeter itself may have some dark current flowing, which should be treated as the actual zero point.

A mercury lamp will be used to illuminate the phototubes. A flashlight, a diffraction grating, a spare color filter wheel, and a red filter are also provided.

Apparatus setup:

1. Turn on the mercury vapor lamp right away so that it can warm up. Close the little slit shield. Aim the lamp so that it does not shine in anyone else's experiment. Do not stare at the mercury lamp. It emits a lot of ultraviolet light which can harm your eyes.

2. Turn on the power supply. Before you hook up the power supply to anything else, use the digital multimeter to set the voltage to approximately 3 V. Do not adjust the power supply

voltage knob again during the experiment. After the power supply is set to 3 V, make all voltage adjustments with the knobs on the photocell box.

3. Carefully examine the circuit diagram on the photocell box and compare it with the diagram below. Make sure that you understand how this circuit works. Find the voltage divider that controls the voltage on the phototube. Where, on the tube, is the current that you will be measuring? And the voltage?
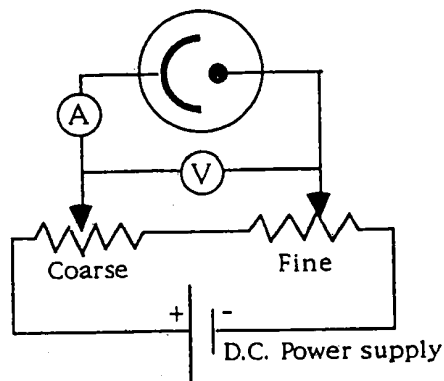

Fig. 3: Wiring diagram of phototube box

4. Set up the photocell box with the apertures. Turn off the power supply, then hook up the circuit as shown on the phototube box. Adjust the measured voltage to zero volt, or reasonably close, using the coarse and fine voltage adjustment knobs on the phototube box. Don't touch the power supply knob.

5. Place the mercury vapor lamp close to the photocell (but not in touch with the photocell because of heat). Position the lamp so that you won't be staring into it and so that it won't be aimed at someone else's apparatus. Open the little slit shield and adjust the lamp so that the brightest light falls on the phototube.

Investigation 1: Does the quantity of ejected electrons depend on the light intensity?

Set the voltage between the cathode and anode to zero, or as close as you can get. Adjust the distance between the lamp and the photocell so that the photocurrent is not more than about 1000 nA. Measure the photocurrent as a function of the AREA of the aperture, in square inches. Graph this function in your lab book. Interpret your graph. The intensity is proportional to the aperture area. Does the photocurrent depend linearly on intensity?

In your lab book: Explain what you did and what you thought about the results. Show your data in table and graph form. Use at least half of the page to make your graph. Interpret your data.

Answer this question in your lab book: Does the quantity of ejected electrons depend on light intensity? Base your answer on your experimental results.

Investigation 2: Does the maximum kinetic energy of the electrons depend on the light intensity?

If you gradually apply positive voltage to the photocathode, the photocurrent (number of ejected electrons) will decrease. Eventually, none of the photoelectrons will have enough energy to escape. The stopping potential V_s is the voltage which just causes the photocurrent to become zero. The maximum kinetic energy of the photoelectrons (in electron volts) is $K_{max} = e\, V_s$, where e is the charge of the electron.

Your job is to measure V_s for each aperture size. Remember, use AREA, not diameter.

In your lab book: Explain what you did. Show your data in table and graph form. Interpret your data.

Answer this question in your lab book: Does K_{max} depend on light intensity? Be sure you base your answer on your experimental results.

Investigation 3: Is there a maximum wavelength (minimum frequency) of the light?

As you may have guessed, you're going to test this by shining different wavelengths of light on the phototube. First, you need to understand what the color filters are doing to the spectrum of light from the mercury lamp.

You can observe the mercury spectrum by closing the slit shield, then using a diffraction grating to see the spectrum. Observe the effect of the color filters on the light from the mercury lamp. Without any filters, the brightest spectral lines are yellow-orange, green, blue, and purple. Now hold a spare color filter wheel between the lamp and the diffraction grating and look through the grating again. Which spectral lines are eliminated by each filter?

You may need to get a fairly "pure" red light in order to test whether long visible wavelengths can eject electrons. Look at the flashlight through the diffraction grating. The spectrum of the flashlight is continuous with no lines. It is strongest in the red. Look at the flashlight's light coming through the filters with the diffraction grating. Using the red filter in combination with the spare color wheel, find the combination that gives the "purest" red light (Fig. 4). In your lab book: Record what you did and what you saw. Note which lines or parts of the continuous spectrum were eliminated by each filter or combination of filters.
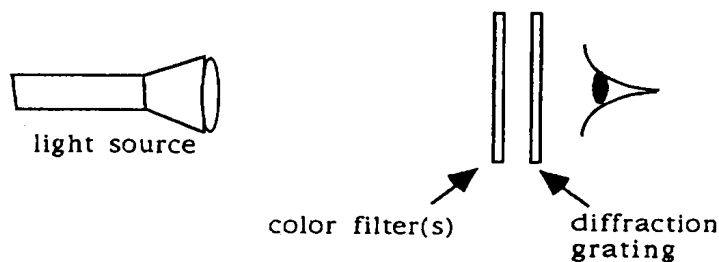

Fig. 4: Obtaining a red light

Now you're ready to complete this investigation. Change boxes so that you are now using the phototube box with the color filters on it. Set the voltage equal to zero using the knobs on the box. Don't touch the power supply! Shine the mercury lamp on the phototube and note the value of the photocurrent, using each filter and the open (no filter) position of the wheel. The filters are labeled according to their cutoff wavelengths of light, in nanometers (1 nm = 10^{-9} m). Recall that frequency = c/λ and that red = long wavelength, purple = short wavelength.

Remove the mercury lamp and shine your "pure' red light (577 nm + red filter) onto the phototube. What happens? Interpret your results.

In your lab book: Record your measurements of photocurrent vs. wavelength. Answer the question: Is there a maximum wavelength (minimum frequency) of the light capable of ejecting electrons? If so, approximately what is its value? Justify your conclusions by referring to your data.

Investigation 4: Does the kinetic energy of the electrons have a maximum value?

Set up the mercury light in front of your photocell, with the slit open. Measure and graph the photocurrent vs. voltage for the case with no color filter. Find the stopping potential (the voltage at which the photocurrent becomes zero).

In your lab book: Explain what you did. Show your data in table and graph form. Interpret your graph.

Answer this question in your lab book: Is there a maximum kinetic energy of the electrons? If so, what is its value?

Investigation 5: Does the maximum kinetic energy of the electrons depend on the wavelength of the light?

Now that you know that the kinetic energy has a maximum, measure K_{max} as a function of wavelength. (Basically, repeat investigation 4, using all of the color filters.) Obtain K_{max} from the stopping potential ($K_{max} = eV_s$) and graph K_{max} as a function of frequency and of wavelength (2 graphs).

In your lab book: Explain what you did. Show your data in table and graph form. Draw this graph as large and carefully as you can, using a full page of your data book. Interpret your data. What does it mean?

Answer this question in your lab book: Based on your data, how does the kinetic energy depend on the wavelength and frequency of the light?

Investigation 6: What is the value of the Planck's constant?

Albert Einstein successfully explained the photoelectric effect as the transfer of kinetic energy between light and electrons. The energy of a particle of light is $E=h\nu$, where ν is the frequency of the light. The proportionality constant h is called Planck's constant, and is one of the fundamental constants in physics: $h=6.63\times10^{-34}$ J s = 4.14×10^{-15} eV s.

The maximum energy of the photoelectrons is

$$K_{\max}= h\nu - W,$$

where W (called the work function) is the amount of energy needed to separate an electron from the photocathode.

To measure h and W, use your data from investigation 5. The equation for $K_{\max}$ is the equation of a line of the form $y = mx + b$ where h and W are the slope and intercept of the line. Read the values of h and W off of your graph of $K_{\max}$ vs. ν. Then take the accepted value for h and your best guess for W, and plot a new line of slope (accepted value of) h and intercept W on the same graph as your data. Label the experimental and theoretical lines.

In your lab book: Interpret your graph. How close is your measured data to the theoretical line?

Answer these questions in your lab book: How well were you able to obtain the value of h? Suggest some ways to get a more accurate value.

Overall Conclusions:

When you have finished all of the investigations, stop and reflect on what you have done. Are you satisfied that the experiments confirm the physical principles under investigation? Why or why not? Is there anything that you would do differently? Are there any lingering questions in your mind? Write a little bit about these things in your lab book. The experiment isn't complete until you have done this.

Emission Spectrum of Hydrogen

I. Introduction

One of the most interesting phenomena in nature is that elemental substances such as sodium, mercury or hydrogen absorb and emit light at a number of well-defined frequencies. These atomic absorption and emission spectra, as they are called, became one of the most important tools used to investigate the quantum mechanical nature of matter at the atomic scale. They were the motivation behind the development of the Bohr model of the hydrogen atom. They continue to be a significant test for the more sophisticated atomic and molecular models used today, and are used in chemistry, astrophysics and form the basis of laser technology.

For elemental hydrogen, H, the spectrum is very orderly, and can be grouped into series of spectral lines, with each series showing the same pattern of lines. The series are labeled $n_s = 1, 2, 3$, etc., in order of increasing wavelength, and the lines in each series can be numbered from $n = n_s+1 \rightarrow \infty$, also in order of increasing wavelength. The Rydberg formula expresses the wavelength of the spectral line as a function of the series and line number.

$$\frac{1}{\lambda} = R\left(\frac{1}{n_s^2} - \frac{1}{n^2}\right), \tag{1}$$

where $R = .01097$ nm^{-1} is the Rydberg constant. The Balmer formula describes the spectral series for which $n_s = 2$.

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{n^2}\right), \tag{2}$$

This series includes the visible wavelengths of light. While these formulas provide an accurate expression of the entire spectrum of hydrogen, they offer no physical explanation for why the spectral lines are distinct, why the wavelength depends on the difference between the two numbers, or what determines the value of the Rydberg constant.

Niels Bohr developed a model for the hydrogen atom that was a variation of the Rutherford planetary model of atoms. In this model, Bohr postulated that the electron in its motion is governed by the electrical attraction with the proton in the nucleus, but that it is restricted to certain circular orbits having quantized angular momentum, $L = n\hbar,$, where n is an integer, and $\hbar$ is the Planck's constant. Although Bohr could not explain why the angular momentum should be quantized in this way, his model went a long way toward explaining the behavior of electrons in atoms.

The Bohr postulate about quantized angular momentum leads to a prediction that the electron in orbit around the hydrogen nucleus is allowed to have only certain, well-defined energies, given by the formula

$$E_n = \frac{E_0}{n^2}.$$

(3)

The orbital energies are labeled from $n = 1$, which is the ground state, or lowest energy level, up to $n = \infty$, which corresponds to a free electron that is not bound to the hydrogen nucleus at all. The lowest energy is the ground state energy, $E_0 = -13.6$ eV. The electron is allowed to have these energies, and no other, while in orbit around the nucleus.

Working from this model, we can explain why hydrogen absorbs light only at certain frequencies, and, when excited, emits light at those same frequencies. As we have shown in the Photoelectric Effect experiment, light of frequency ν consists of photons, labeled γ, each of which has the energy $E_\gamma = h\nu$. Under most circumstances, the electrons in a gas can absorb only one photon at a time. If the electron is initially in the state $n = 1$, the photon it absorbs must give it an energy corresponding to the energy of the states $n = 2$, 3, etc.. Otherwise, the photon cannot be absorbed at all. Therefore, the photon energy $E_\gamma = h\nu$ must be equal to ΔE_{1n}, the energy difference between the 1st orbital energy and the nth orbital energy. If the electron in the atom is excited to a higher energy level by some means, it will eventually give up its energy as a single photon with energy equal to $\Delta E_{nu,nl}$, the energy difference between the higher energy level, n_u, and the final energy level, n_l. The different series of spectral lines correspond to the set of all possible transitions to each of the different final states. For instance, the Balmer series in hydrogen involves all the possible transitions down to the $n = 2$ orbital state. A massive number of such transitions are responsible for the light we see from the hydrogen vapor lamp.

In this experiment, we will use a diffraction grating and a spectrometer to resolve the hydrogen light into its visible spectrum, and determine the wavelength of the spectral lines as a function of the initial orbital energy level, n_u. The visible spectral lines are part of the Balmer series, for which we know that $n = 2$, and we will plot $1/\lambda$ vs. $1/n_u^2$ to verify that this is a linear relationship, and also predict the value of R, the Rydberg constant. In doing so, we will have indirectly demonstrated the quantum mechanical nature of the electronic behavior in hydrogen. We will also use one of the spectral lines from the mercury vapor lamp of known wavelength to verify the grating spacing guaranteed by the manufacturer.

II. Apparatus and procedure

A picture of a typical student spectrometer is shown in Fig. 1. The one you are going to use in the lab may be slightly different. A diagram of the light path in the spectrometer is shown in Fig. 2.
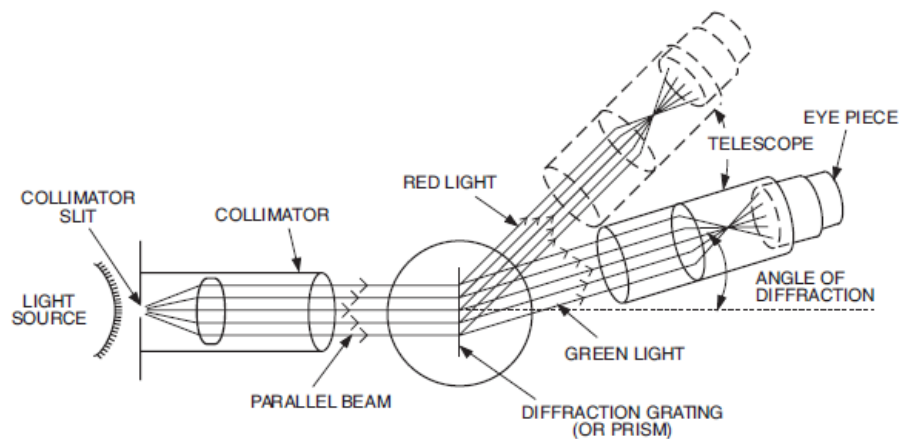
Fig. 1 A Spectrometer

Fig. 2 Diagram of a spectrometer

When using the spectrometer special care must be taken to reduce as much as possible any extraneous light so that all four visible lines in the hydrogen spectrum can be seen. It is a common mistake to identify the strong blue-violet line as the much weaker violet line. You may need to increase the slit width in order to observe the violet line. Remember, however, that the wider the slit, the greater the error in determining the diffraction angle. In general, you should use the minimum slit width at which the spectral line can be observed in order to obtain precise and accurate results.

Make sure that the diffraction grating is placed on the center stand so that the light from the telescope is incident normally upon the grating. For normal incidence, the positions of the spectral lines are given by

$$m\lambda = d \sin\theta, \tag{4}$$

where m is the spectral order, λ is the wavelength of the spectral line, d is the distance between rulings on the grating, i.e., the "grating constant" and θ is the diffraction angle. The grating has a spacing of 600 lines/mm.

Question 1: Derive the expression $m\lambda = d \sin\theta$ for light passing through two slits separated by a distance d using Fig. 3. Assume that $d \ll L$, the distance from the diffraction grating to the point

of measurement. You should calculate the path difference for light passing through the two slits and then impose the condition for when the light will interfere constructively.
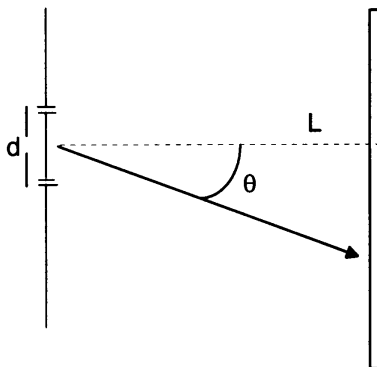

Fig. 3 Light diffraction from a grating

The angle θ is measured from the $m = 0$ spectral line. However, in order to decrease the percentage error of our measurement of the angle, we will measure the angle for the $m = 1$ spectral line of the same wavelength λ both to the right and to the left of the $m = 0$ line and take the difference between the two recorded angles. Dividing by 2, we then obtain θ, but with an error ½ as large as if we had measured θ from the $m = 0$ line.

III. Determination of the grating constant

Each year the spectrometers used in this lab can be somehow different. There are three tricks which can help us to observe the spectral lines more clearly. 1) If necessary use double-sided tapes to secure the spectrometer on the table. 2) Adjust the slit width. 3) Finely shift the lamp left and right so that the spectral lines appear the brightest.

Now, using a line of known wavelength, let's test the information given by the manufacturer about the grating. As a standard for calibrating, use one of the strong lines of mercury (See the photoelectric effect experiment). Watch out that you use a singlet line (the 546.1 nm green one, for instance), and not a doublet (the strong yellow line). Take the first- and second-order readings on both sides of the zero-order, and from these determine the grating constant d and the number of grooves per unit width. As usual, express your result to no more significant figures than are justified by your data. Compare your result with that obtained from the manufacturer's value given on the grating. Use a table as follows:

Line-color from mercury spectrum		
λ (nm)		
Spectral order m	1	2
Reading (deg.) – right		
Reading (deg.) – left		
Difference (=2θ) (deg.)		
θ (deg.)		
$\sin\theta$		
Grating constant d (nm)		
Grating lines per mm (experimental)		
Grating lines per mm (manufacturer)		

Make sure that you record the error in your measurements and calculate the error in your experimentally determined value of d. With due consideration to the uncertainty in your result, does your result agree with that of the manufacturer? If not, what is a likely reason for the difference?

IV. The emission spectrum of atomic hydrogen

Now, replace the mercury lamp with the hydrogen gas lamp. A new hydrogen lamp emits pinkish light. Measure 2θ for m = 1 for each of the four visible lines in the spectrum. The short wavelength lines will be closest to the m = 0 line. Calculate the wavelengths of the lines you can see, using the formula given in part II. Then plot $1/\lambda$ vs. $1/n_u^2$, using your experimentally obtained values of θ and the value of ν given in the following table for each line. What shape is the curve expected to exhibit? Use a table as follows.

Hydrogen line-color	Red	Blue-Green	Blue-Violet	Violet
Grating constant d (nm) (Use measured results.)				
Reading (deg.) – right				
Reading (deg.) – left				
Difference (=2θ) (deg.)				
θ (deg.)				
$\sin\theta$				
λ (nm) (experimental)				
For graphing: Quantum number n_u.	3	4	5	6
$1/{n_u}^2$				
$1/\lambda_{exp}$				

Use the Balmer equation to show that the *y*-intercept is $R/4$. Find R from your graph, then use it (*a*) to find the short wavelength limit of the Balmer series, and (*b*) in conjunction with the Rydberg equation and the Einstein relationship $E_{photon} = hc/\lambda$ to determine E_I, the ionization energy of hydrogen which is required to remove an electron in the ground state. In a table, compare the two results with the published values.

Question 2: Why can't we see the transitions to the $n = 1$ ground state?

Question 3: What is the mechanism in the hydrogen lamp which causes the hydrogen atom electrons to be in the higher energy orbital states from which they fall, creating the spectral line pattern you see through the diffraction grating?

Measurement of the *q/m* Ratio of the Electron

Measuring the charge-to-mass ratio of charged particles is fundamental to our understanding of physics. This ratio allows high energy physicists to identify different kinds of particles. The *q/m* ratio is used to sort ions in mass spectrometers, which are used for chemical analysis. Historically, the *q/m* ratio was used to prove that alpha and beta rays consisted of two different kinds of particles.

When a charged particle enters a magnetic field, the force on the particle is given by:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}. \tag{1}$$

When the velocity is perpendicular to the magnetic field, the force is perpendicular to the velocity and is therefore a centripetal force. The motion of the particle is circular and is described by the equation

$$F = \frac{mv^2}{R} = qvB. \tag{2}$$

If you solve this equation for *R*, you will see that the radius of the circle *R* depends on the momentum, charge, and mass of the particle. If the kinetic energy of the particle is $K = \frac{1}{2}mv^2 = qV$, where *V* is an accelerating voltage, we obtain the equation

$$\frac{q}{m} = \frac{2V}{B^2 R^2}. \tag{3}$$

This equation is very useful. While one cannot use it to measure *q* or *m* independently, it is often enough to measure the ratio *q/m*. Chemists use *q/m* to identify ionized atoms vaporized from an unknown sample in a mass spectrometer. High energy (particle) physicists use *q/m* to identify exotic subatomic particles. Atomic physicists use *q/m* to determine how many electrons are missing from an ion. Nuclear physicists use *q/m* for nuclear isotope identification and separation. The *q/m* ratio has even been used to sort bulk powders or pellets by particle size.

You will be measuring *q/m* for the electron because the electron is the most easily produced charged subatomic particle. In this experiment you will observe electrons as they circle in a magnetic field and then measure the radius of their path in order to get *q/m*. Finally, you will compare your result with the accepted value given in your text.

Apparatus:

Fig. 1 *e/m* apparatus

The apparatus for the experiment is shown above. A small electron gun is placed inside a large vacuum tube which is filled with hydrogen gas at a pressure of 10^{-2} torr. (1 torr = 1 mm Hg; 1 atmosphere = 760 torr.) As the pressure in most vacuum tubes is 10^{-7} torr or less, this tube has a very high gas pressure for a vacuum tube. The electrons collide with and ionize the gas in the tube, causing the path of the electron beam to be visible.

To do the experiment properly, you need to know how the electron gun works: When a wire is heated, it becomes surrounded by a cloud of electrons. If you put a negative charge on the wire and a positive charge on a nearby electrode (the anode), the electrons are accelerated. The anode has a hole in it to let the electron beam go through. The kinetic energy of the electrons depends on the voltage difference between the filament and the anode, which you control. Between the filament and the anode is the Wehnelt cylinder, as shown in the figure below, which is an electrode that helps to focus the beam of electrons The Wehnelt cylinder voltage should be zero for this tube.
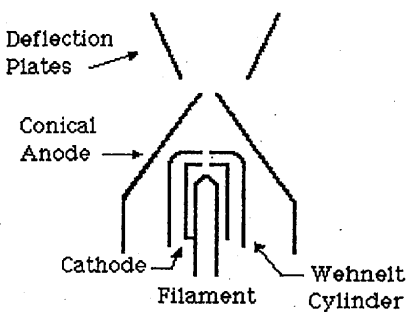
Fig. 2 Electron gun configuration

The cathode ray tube is surrounded by a pair of Helmholtz coils. Helmholtz coils are two parallel circular coils of wire that are separated by a distance that is equal to their radius. This geometry causes the magnetic field to be nearly uniform anywhere in the region between the coils. The equation for the *B* field of Helmholtz coils is:

$$B = \frac{\mu_0 \left(\frac{4}{5}\right)^{3/2} nI}{a}.$$ (4)

Here n is the number of turns, I the current, and a the radius of the coils.

Before you get into the laboratory:

Please answer these questions:

Question 1: Try to derive Eqs. 3 and 4. Show the derivations in your lab book. Hint: You can find this derivation already worked out in many introductory physics textbooks. Look under "Magnetic Field from a Circular Coil of Wire." If you refer to or copy from a book, give its bibliographic reference.

Question 2: Use Eq. 4 to calculate the expected B, in Teslas, for your magnet using magnet currents of 1, 2, and 3 amperes. For these coils, $n=130$ turns and $a = 15$ cm. The constant $\mu_0 = 4\pi \times 10^{-7}$ N/A^2. Notice that in Eq. 4, $B = cI$ where c is a proportionality constant. If you figure out the value of this constant ahead of time and write it in your book, you'll save a lot of time when you get into the laboratory.

Question 3: Compare the size of the earth's magnetic field (1/4 Gauss $\approx 2.5 \times 10^{-5}$ T) with your answers to question 2 above. Is the earth's field going to affect your experiment? Is there a simple way (there is!) to minimize the effect of the earth's field? (Hint: think about vectors.)

Question 4: Starting from Eqs. 3 and 4, derive an equation for the diameter of the electron beam circle as a function of magnet current.

Apparatus setup:

We recently replaced some *e/m* tubes and their control units which have served our department for tens of years. For the power supplies we used the original ones from Leybold, a German company. Power supplies are relatively durable and fairly expensive electronic devices.

Hook up the apparatus as follows. DO NOT TURN ANYTHING ON UNTIL THE INSTRUCTOR HAS CHECKED THE CIRCUIT! The tubes can be damaged if the circuit is wrong. They're very expensive.
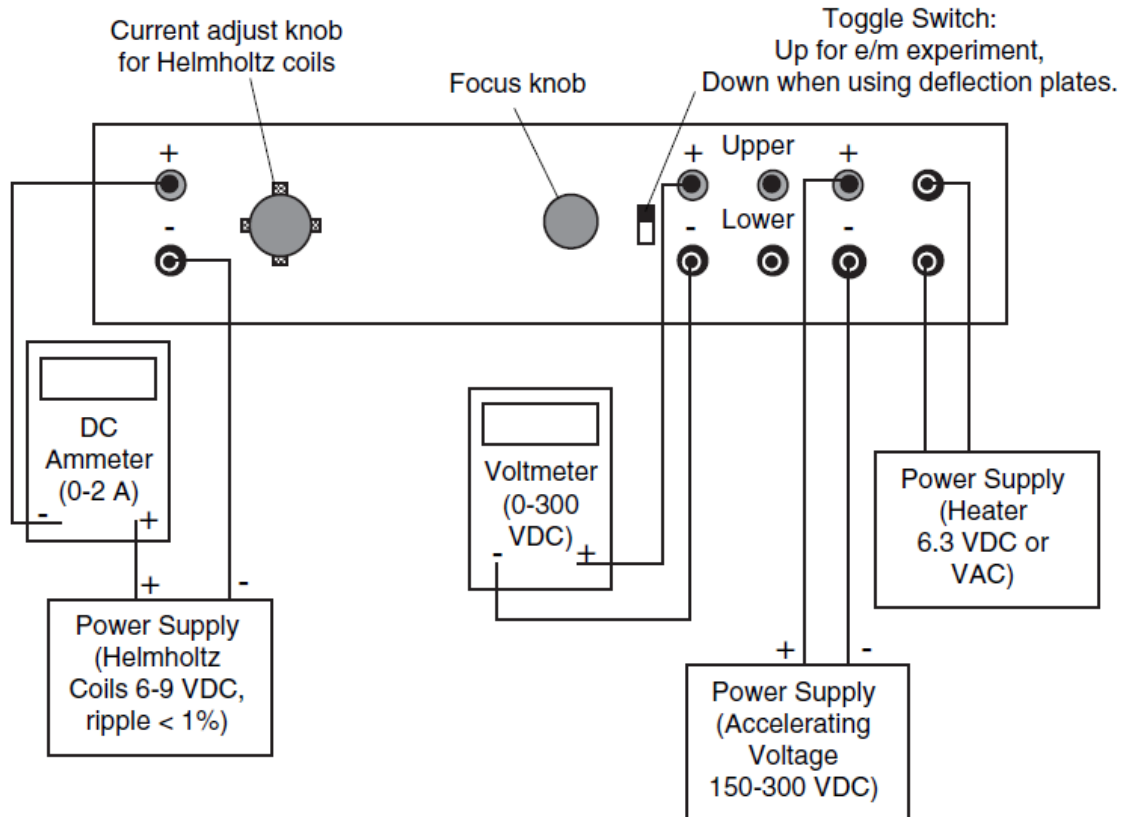
Fig. 3 Circuit connection for the *e/m* experiment

For the voltmeter and ammeter, use the digital multimeters (DMMs). Make sure that the input leads for the "ammeter" are connected in the correct places on the DMM, and that the buttons on the DMM's are pushed to get the meters into the correct mode. The current in the magnet coils will be **between 1 and 3 amps**. The anode voltage must be **between 150 and 250 volts**.

After the instructor has checked the circuit, you may begin the experiment. Make sure that the knobs on the Leybold power supply are turned all the way down and check to see that the "380 V' switch is in the up position. Turn on the power supply. As soon as the filament is glowing, turn the anode voltage up to 150 volts or so. You should see a blue beam coming out of the electron gun. You may need to turn off the ceiling lights.

Turn on the magnet. If the electron beam bends around in a circle, you're fine. If the electron beam goes the wrong way, turn the magnet off and switch the polarity of the magnetic field wires. If the electron beam winds into a spiral instead of a circle, rotate the tube in its brackets and/or rotate the apparatus on the table until you get a circle.

Using the virtual-image projector:

These are handy little gizmos that let you measure the radius of the electron beam by sticking the virtual image of a ruler inside the tube! The rulers on the virtual image projectors are divided into millimeters. The gap in the center of the ruler is 2 cm wide. Set the projector up like this:
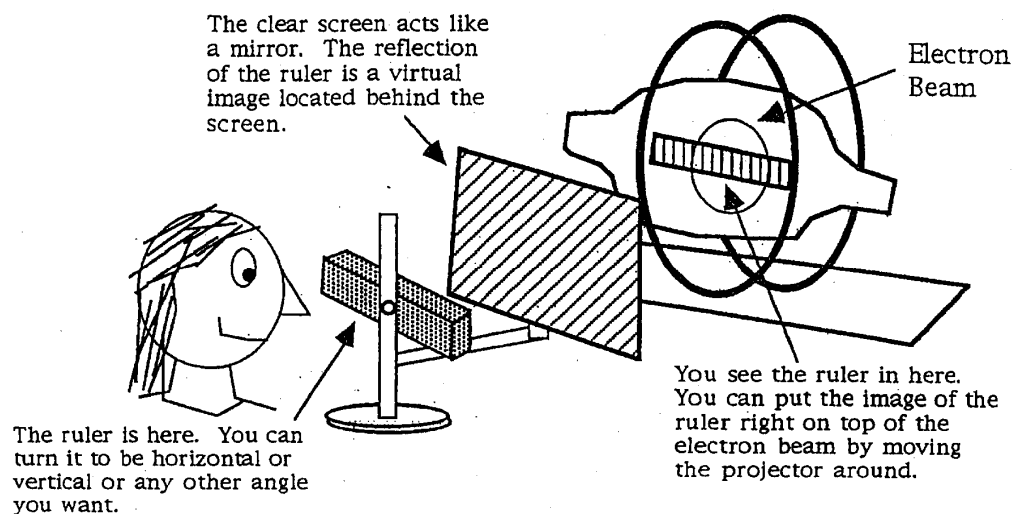

The clear screen acts like a mirror. The reflection of the ruler is a virtual image located behind the screen.

Electron Beam

The ruler is here. You can turn it to be horizontal or vertical or any other angle you want.

You see the ruler in here. You can put the image of the ruler right on top of the electron beam by moving the projector around.

Fig. 4 Using the virtual-image projector

Investigation 1:

Get the electron beam going into a nice circle. Measure the circle with your scale-projection apparatus. The electron beam spreads out as it travels. Use the outside edge of the beam when you measure the radius.

Question 5: Is the beam path circular, or a little egg-shaped? By how much? Be quantitative. Give an answer in millimeters and in % deviation from the average radius. This is a quick check on a possible source of experimental error. The beam path should be nearly circular. If it isn't, make sure that you keep the angle of your virtual image ruler constant in subsequent measurements.

Investigation 2:

Measure the diameter of the electron beam circle as a function of magnet current for a voltage near 200 V. Start from the minimum current that produces the largest circle. Increase the current by about 0.2 A/step and measure the diameters of the circles. Measure at least 4 different currents. Do this carefully.

Question 6: Does the equation that you derived in Question 4 predict the experimental curve correctly? To answer this question, plot your measured diameters on the vertical axis of a graph and the magnet current on the horizontal axis. Then calculate (using the equation derived in Question 4) the diameter of the beam path for a few values of the current and plot the calculated values on the same graph as your experimental data. Draw a curve through the calculated points,

but not the experimental data. Comment on how well the theory and data match, or why they don't match. You must use words and sentences to discuss your graph.

Investigation 3:

Measure and plot the diameter of the circle as a function of the anode voltage, using a fixed magnetic field at a current of about 1.5 A. Keep the anode voltage between 150 and 250 V, and use about 20 V/step.

Question 7: Does Eq. 3 correctly predict the experimental results? Support your answer by referring to your data. (Hint: Treat q/m as a constant, and consider doing an analysis similar to that in Question 6 above.)

Investigation 4:

This may be completed outside of lab, as it involves discussing and thinking. Use your data to find the best value for q/m that you possibly can. Discuss and compare your results and experimental errors with other laboratory groups. Record these discussions in your laboratory notebooks, giving full credit for ideas where credit is due. Example: "Mortimer suggested that the spread of the electron beam could change the results by 3%." Finally, compare your best value with the accepted textbook value and discuss any major differences,

Additional Questions:

Please answer these questions briefly in your laboratory notebook.

Question 8: Why does the beam spread out as it travels in the tube? Back up your answer with some physical reasoning.

Question 9: Why do you get a spiral pattern when the tube is rotated? Hint: Think about the vector components of the initial electron velocity interacting with the magnetic field.

Question 10: Calculate the speed of the electrons for the minimum and maximum anode voltages used. Express the speed in meters/second. Hint: These electrons are not relativistic.

Question 11: (Optional, up to 10 pts. extra credit) Read up on gas discharges and find out why the electron beam appears blue. When you find out, please tell the rest of us.

Electron Diffraction

The fact that electron diffraction occurs at all conclusively proves that electrons are waves. In this lab you will be using a beam of electrons to produce diffraction patterns using crystals of either aluminum or graphite as the diffraction "gratings." You will learn, how to analyze the diffraction patterns in order to measure the distance between the atoms in these crystals.

The apparatus is fragile and decays with use. Do not turn on the apparatus until you have read and understood the preliminary theory and discussion and are ready to take data.

Theory:

Diffraction is a wave phenomenon that occurs whenever a wave passes through two or more slits. The slit width must be comparable to the length of the incoming wave. For water waves in a ripple tank, diffraction patterns can be produced by placing barriers in the water several millimeters apart. For visible light the slits of the diffraction grating need to be a few hundredths of a millimeter wide. For x-rays and electron waves the diffraction grating is formed by the layers of atoms in a crystal. In every case, the size of the slits must be comparable to the size of the wavelength.

Like photons, electrons and other subatomic "particles" exhibit both wave and particle characteristics. The French physicist Louis de Broglie argued that electrons and photons should obey the same equations that relate energy and momentum to frequency and wavelength. Experimentation has shown de Broglie's hypothesis to be correct, and therefore for electrons we can use

$$p = \frac{h}{\lambda} \tag{1}$$

to determine the wavelength (called the "de Broglie wavelength") of the electron.

_____

Example: What is the wavelength of an electron with kinetic energy 50 keV?

The kinetic energy K of this electron is 50 keV. The total energy is kinetic energy plus the rest energy, so that

$E = K + mc^2 = 50$ keV + 511 keV = 561 keV.

The momentum of this electron is given by

$$p = \frac{1}{c}\sqrt{E^2 - mc^2} = \frac{1}{c}\sqrt{(561\,\text{keV})^2 - (511\,\text{keV})^2} = 231.5\,\text{keV}/c.$$

The de Broglie wavelength of the electron is therefore

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1240\,\text{eV} \cdot \text{nm}}{231.5\,\text{keV}} = 5.36 \times 10^{-3}\,\text{nm} = 0.054\,\overset{\circ}{\text{A}}.$$

Note that this wavelength is much smaller than the size of an atom (approximately 1 Å).

Now let us look at the phenomenon of diffraction. The following figure represents a wave being reflected from a series of crystal planes. For constructive interference to occur, the difference in the length of the two paths must be $n\lambda$, where n is an integer representing the order of the diffraction pattern. Otherwise, destructive interference occurs and no reflected wave is seen.
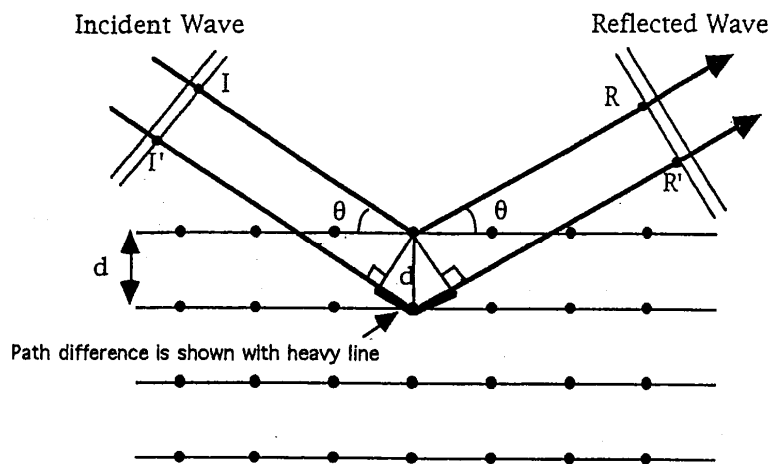
Fig. 1 Geometry for Bragg diffraction

Exercise 1: Do a mathematical proof, based on Fig. 1, using geometry to show that the path difference $n\lambda$ is equal to

$$n\lambda = 2d \sin\theta \tag{2}$$

When we use actual crystals, we find that there are many possible values for the plane separation d. The number of possible values depends on the arrangement of atoms in the crystal structure and the tilt of the crystal with respect to the incident electron beam. Fig. 2 shows how to tilt a crystal to get three different values of d.
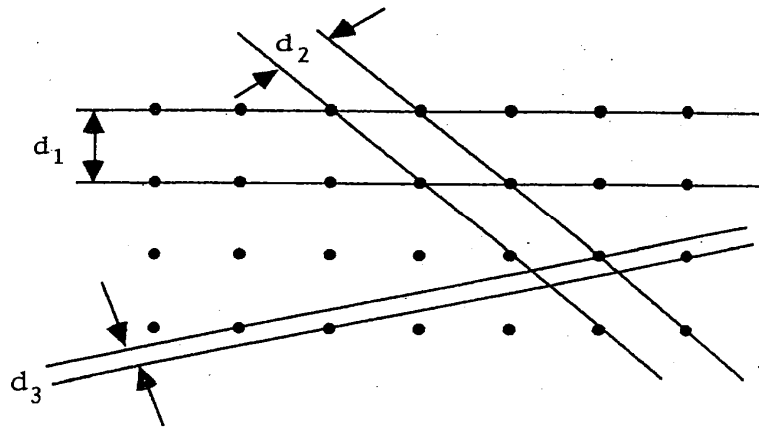
Fig. 2 Crystal orientation and plane spacing

Although Eq. 2 is simple, this variation in the plane spacing can make the experimental results complicated to interpret. To analyze our diffraction patterns, we need to consider the geometry of the apparatus and relate it to the Bragg diffraction geometry.

Apparatus:

The apparatus is shown in Fig. 3. It consists of a specially-built cathode ray tube. The electron gun in the neck of the tube shoots a beam of electrons towards an internal target. A set of electrostatic deflectors allows you to aim the beam at any point on the target. When the electrons strike a crystal or set of crystals in the target, the diffraction pattern appears on the screen at the front of the tube.
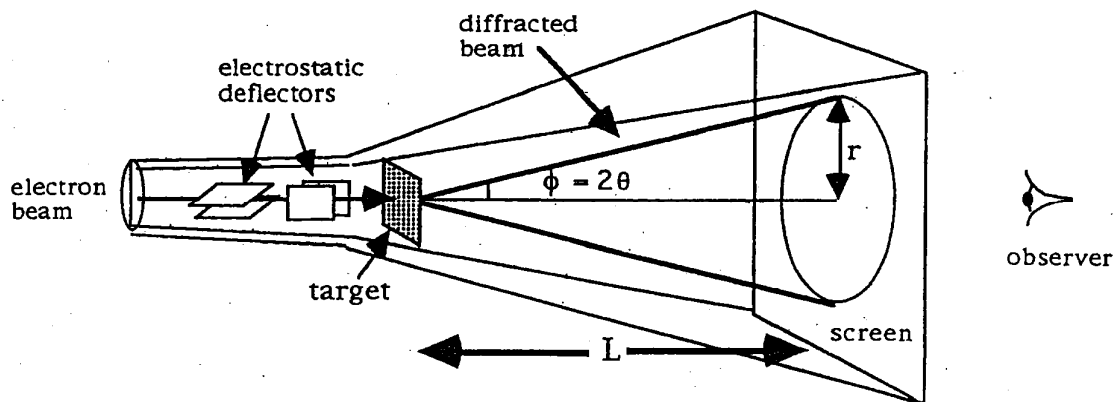
Fig.3 Geometry of the apparatus

The target contains both aluminum and graphite samples which are made of very thin films covering different quadrants of the target support grid. The target materials, like many everyday materials, are made of microscopic crystal grains that are packed tightly together. The crystals in the aluminum are very small, so that the electron beam strikes many randomly oriented crystals at once. Because of the large number of crystals, the electron beam is diffracted into a set of concentric cones. The "bottoms" of the cones appear as concentric circles on the CRT screen.

7-3

The crystals in the graphite sample are much larger. It is possible for the electron beam to focus on one individual graphite crystal, and in this case you will see a pattern of distinct spots on the screen. When you do the experiment, you will be measuring the radii of these rings and the distances between spots.

The wavelength of the electrons is an important parameter in this experiment. You will control this parameter by adjusting the accelerating voltage in the tube. The voltage determines the kinetic energy (and hence the momentum and hence the wavelength) of the electrons. The kinetic energy of these electrons will be between 5 and 7 keV.

Exercise 2: Are 7 keV electrons relativistic? Why or why not? Give a quantitative argument. (That means you must do the calculation.)

Exercise 3: Use the equations for electron kinetic energy and de Broglie wavelength to derive the following expression for the wavelength of the electrons, where λ is in Å and V is in volts:

$$\lambda = \sqrt{\frac{150}{V}} \; \overset{\circ}{A} . \tag{3}$$

The number 150 has units of $Å^2V$, so the answer will have the proper units. When you derive this equation, be very careful of units so that you don't miss any conversion factors.

Finding *d*:

In your experiment you will be using graphite crystals and polycrystalline aluminum as your diffraction gratings. To find the parameter *d* in Equation (2), you need to know some things about crystals.

The aluminum film in your apparatus is made of many microscopic "grains." Each grain is a single crystal. The metal is made of many single-crystal grains stuck together in random orientations.
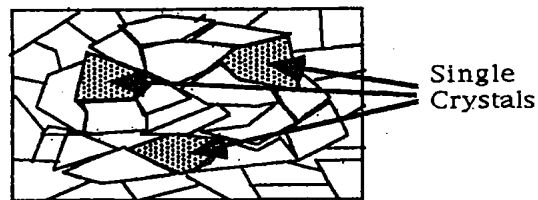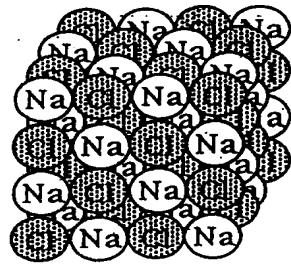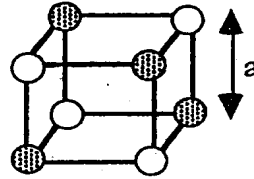

Fig. 4 Grain structure of polycrystalline aluminum

The following discussion shows you how to obtain the various possible values of the plane spacing *d* for a crystal. Crystals are made of regular arrays of atoms. The arrangement of atoms in a crystal of table salt is pictured below:

Fig. 5 Arrangement of atoms in a sodium chloride crystal

A *crystal lattice* is the three-dimensional pattern of atoms making up the crystal. The *unit cell* is the basic three-dimensional "building block" of atoms that makes up the crystal. The lattice is made by stacking zillions of unit cells together. For many crystals (examples: NaCl, aluminum) the unit cell is a cube. Other crystals have different shapes of unit cells:
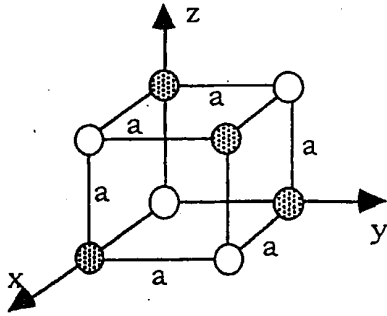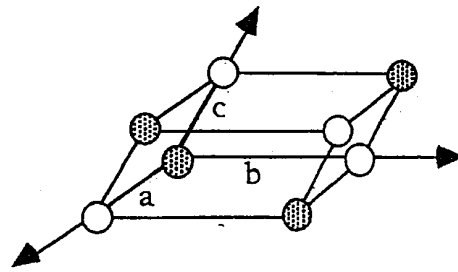

Fig. 6a A cubic unit cell   Fig. 6b A non-cubic unit cell

The *lattice constant "a"* of the cubic unit cell in Fig. 6a is simply the length of any edge of the cube. Any unit cell can be described by a maximum of three lattice constants and three *lattice vectors*. The unit cell in Fig. 6b has three lattice constants a, b, and c, all of which are different. The three lattice vectors are mutually perpendicular in Fig. 6a, but not in the crystal In Fig. 6b. The unit cell, lattice constants, and the angles between lattice vectors are determined mostly by chemical composition. We will not further consider the type of cell pictured in Fig. 6b.

The distance between two planes of atoms in a crystal gives d, the diffraction grating spacing. As you can see from Fig. 2, it is possible to slice a crystal several ways. A real crystal will give us several values of d, depending on its orientation with respect to the electron beam. To find the possible plane spacing d we need to use three-dimensional geometry.

From Fig. 6a it is apparent that the three lattice vectors are $\mathbf{a} = a\,\mathbf{i}$, $\mathbf{b} = a\,\mathbf{j}$, and $\mathbf{c} = a\,\mathbf{k}$ for a cubic crystal. Now consider a possible plane as it cuts through the cube in Fig. 7:
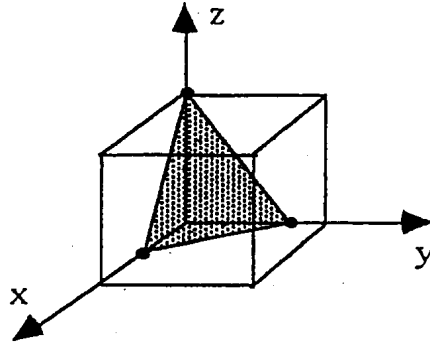
Fig. 7 A plane cutting the cube

The plane in Fig. 7 cuts the coordinate axes at coordinates 1/2 **a**, 2/3 **b**, and 1 **c**. The *Miller indices* of this plane will allow us to calculate *d*, which is the distance between the layers of atoms that form our diffraction grating. The general procedure for finding the Miller indices of a plane is:

1. Find the intersections of the plane with the lattice vectors.

2. Take the reciprocals of the intercepts.

3. Find the smallest set of integers with the same ratios.

In this example the reciprocals of these intercepts are 2, 3/2, and 1. The smallest set of integers with these ratios is 4, 3, and 2. These are called the miller indices (*hkl*) of this plane, and are generally written as (432). Some more examples are found in Fig. 8, along with their Miller indices:
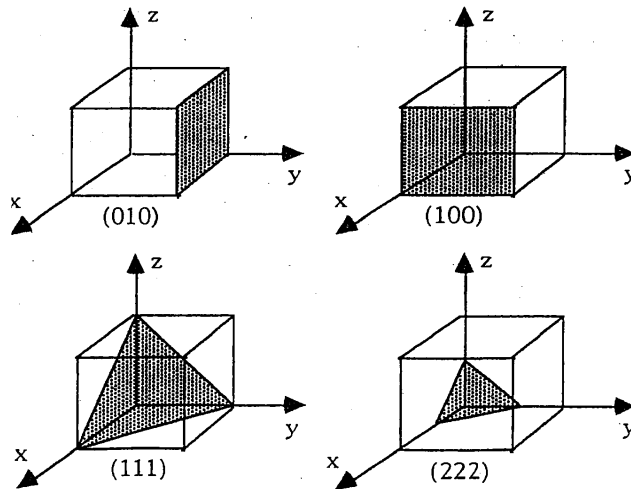

Fig. 8 Miller indices for different planes cutting the cube

Exercise 4: Verify the Miller indices in Fig. 8 by applying steps 1, 2, and 3 described above to the four planes shown in the figure.

A crystal is made by repeating the unit cell pattern over and over. For a cubic lattice the distance between adjacent parallel planes is given by

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}},$$ (4)

where a is the lattice constant, and h, k, and l are the Miller indices. (You're not required to prove this, but you might like to try it for fun.) Substituting this equation into the diffraction equation (Eq. 2), we have

$$n\lambda = \frac{2a \sin \theta}{\sqrt{h^2 + k^2 + l^2}}.$$ (5)

This is the main equation for diffraction using a cubic crystal as a grating.

The crystal structure of aluminum is actually FCC (face-centered cubic). A picture of this structure is given in Fig. 9.
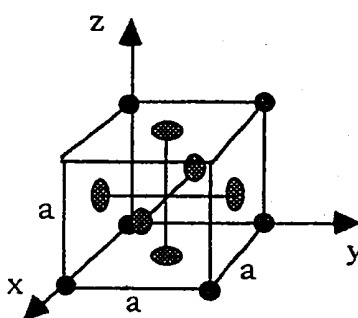

Fig. 9 A FCC (face-centered cubic) crystal

In the FCC unit cell each cube has an atom at the center of each face as well as at the corners. Note that there are planes parallel to the faces of the cube and halfway between them. The electron waves reflected from the opposite cube faces will be out of phase with the waves reflected from the intermediate plane between these faces, and therefore no diffraction will be seen from the (100), (010), or (001) directions. Although we won't present the details here, it turns out that for an FCC crystal no diffraction will occur unless h, k, and l are **either all odd or all even**. The innermost ring of your observed diffraction pattern will correspond to the orientation with the smallest allowed value of $h^2 + k^2 + l^2$, which is (111). The sequence continues with (200), (220), (311), (222), (400), (331), and so forth. Some rings may appear as "double", but they are really two separate rings due to two different crystal orientations.

Exercise 5: Make a table of possible values for (*hkl*) that diffracts the electrons for a FCC cubic lattice, as well as the corresponding values of $\sqrt{h^2 + k^2 + l^2}$. Arrange the Miller indices (*hkl*) in the order of ascending $\sqrt{h^2 + k^2 + l^2}$ and verify the sequence of values given above.

You will be measuring the radii of the ring patterns on the screen, then using geometry to find the diffraction angles. For small angles the amount of deflection of the beam may be approximated as $\sin \theta = \theta$. From the tube geometry shown in Fig. 3, the diffraction angle is $2\theta \approx \tan 2\theta = r_{hkl} / L$. Substitution into Eq. 5 gives

$$a = \frac{n\lambda L \sqrt{h^2 + k^2 + l^2}}{r_{hkl}}, \tag{6}$$

which relates r_{hkl}, the radius that you will measure, to the crystal lattice constant a.

Part 1: Diffraction from polycrystalline aluminum

The goal: Determine the lattice constant a (equivalent to measuring the distances between atoms) for polycrystalline aluminum based on your measurements of the diffraction patterns and your calculations of the electron wavelength. A picture of the apparatus is shown below.

Fig. 10 The electron diffraction apparatus used in our lab

Procedure:

• Look at the cathode ray tube through the window in the side of the apparatus. Identify the deflectors and the target. Find and record the value of L (the target-to-screen distance) for your particular cathode ray tube. This is written on a label somewhere on or inside the apparatus.

• Attach the microammeter to the back of the electron diffraction apparatus.

• Turn the intensity and high voltage knobs all the way down (full counterclockwise).

• Turn on the apparatus.

• Set the high voltage at 6.00 kV, as exact as possible, and adjust the intensity until you see either a spot on the screen, or else the shadow of the target. DON'T LET THE TARGET CURRENT ECCEED 10 MICROAMPS.

• If you see the shadow of the target, the beam is either severely defocused, or it is off the target entirely. Try defocusing and/or moving the beam to see the shadow of the target, then move and/or focus the beam so that you obtain a diffraction pattern of concentric rings on the screen. DO NOT LET THE UNDIFFRACTED BEAM SPOT SIT STILL ON THE SCREEN

OR YOU CAN BURN THE SCREEN. The pattern should resemble the following picture. You need to turn off the ceiling lights and use your flashlight for operation of the apparatus.

Fig. 11 The diffraction pattern from polycrystalline aluminum

• Quick check: Measure several different diameters of one ring to check for distortion of the image. If your measured diameters are within 1 or 2 mm of being equal, you don't have to worry about distortion anymore. If the different diameters are not sufficiently equal, you'll have to eliminate this source of experimental error with a suitable averaging procedure.

• Once you have obtained nicely focused rings, measure the diameters of the rings with either the calipers or the clear plastic ruler. Record the results in a table and sketch the rings in your lab book. Sometimes you can only see part of a ring. In this case, measure it as best as you can. A pair of closely spaced concentric rings counts as two separate rings.

• Test the diffraction patterns using another voltage, e.g., 8 kV. The electron beam will shift position when you change the voltage, so you will have to bring it back to a place where diffraction rings may be found.

Exercise 6: Explain why the beam should shift position when you change the voltage. Yes, the energy of the beam changes, but why should this make a difference? What, specifically, is happening to the beam inside of the tube?

Data analysis for polycrystalline aluminum:

• Collect your data and results in a table and discuss your findings. Figure out exactly which crystal plane (or set of planes) is associated with each ring. Do this by listing the combinations of Miller indices that could possibly give you each ring.

Example: You have two rings, labeled A and B. The ratio r_A/r_B can be used to check your guesses for the Miller indices by the following procedure:

First, note that for this geometry, we have $\sin \phi \approx \tan \phi = \dfrac{r}{L}$. Verify that you are justified in using this approximation by plugging in the values of r and L for your largest ring. How big is the error that you get by using this approximation? Why should you use your largest ring in this calculation?

Next, substitute r/L into the Bragg diffraction equation, and divide the equation for ring A by the equation for ring B:

$$\frac{n_A \lambda}{n_B \lambda} = \frac{\dfrac{2a \sin \phi_A}{\sqrt{h_A^2 + k_A^2 + l_A^2}}}{\dfrac{2a \sin \phi_B}{\sqrt{h_B^2 + k_B^2 + l_B^2}}}. \tag{7}$$

Note that a is constant because it's a property of the target crystal, L is constant because It's a built-in property of the apparatus. Also, λ is a constant as long as you haven't changed the accelerating voltage of the electron beam. Now Eq. 7 can be simplified, using the small-angle approximation $\sin \phi \approx \tan \phi = \dfrac{r}{L}$ and assuming $n_A = n_B$.

$$\frac{r_A}{r_B} = \frac{\sqrt{h_A^2 + k_A^2 + l_A^2}}{\sqrt{h_B^2 + k_B^2 + l_B^2}}. \tag{8}$$

Now, if your guesses for the Miller indices were (200) and (220), Eq. 8 becomes

$$\frac{r_A}{r_B} = \frac{\sqrt{2^2 + 2^2 + 0^2}}{\sqrt{2^2 + 0^2 + 0^2}} = 1.41.$$

If this matches r_A/r_B, calculated from your measured diameters of these two rings, you have guessed the correct Miller indices. If the match is poor, try other combinations of (hkl) until you get a reasonable match. Keep in mind that it's possible for you to miss a faint ring. This method of analysis will catch such errors.

   • Once you have identified the correct sets of Miller indices for your observed rings, sketch figures showing the crystal planes that are producing the rings that you see. These sketches should be similar to Fig. 8 above. Be quantitative when you do this. Use the Miller indices to figure out where the intercepts are.

   • Once you know the Miller indices, calculate the lattice constant a for aluminum using your data. Calculate the value of a for each voltage and compare it with the accepted value of $a=$ 4.04958 Å obtained from x-ray diffraction measurements.

   • Do an error analysis. Note that this is "error," not "mistake." Estimate quantitatively the quality of your measurements. How accurately could you determine a? How much uncertainty did you have in your measurement, in Ångströms? Where could this error have reasonably come from, and how much could each source of error contribute to the total error? Hint: Look at how changing the voltage changes the overall size of the rings. You may want to graph lattice constant vs. voltage and play with your data to look for a systematic error.

Part 2: Single-crystal diffraction using pyrolytic graphite

Unlike a polycrystalline mass which produces diffraction rings, a single crystal placed in the electron beam will produce a set of spots on the screen. The number, intensity, and pattern of the spots depend on the internal symmetry properties of the crystal and the order n of the diffraction. The unit cell of graphite is hexagonal, as in Fig 12.
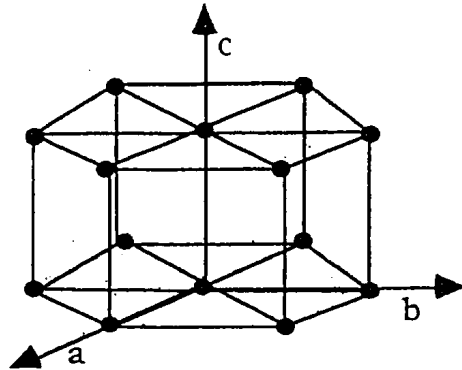

Fig. 12 Hexagonal crystal structure

The hexagonal crystal is most easily described by using the **a**, **b**, and **c** axes shown in the figure. Note that the **a** and **b** axes are 120 degrees apart. Pyrolytic graphite is a polycrystalline form of carbon with strong preferred orientations. Its structure can be thought of as a stack of single crystals which are randomly oriented about the **c** axis but with a preference for parallel **c** axes. Since 6 kV electrons will penetrate the target to only a few hundred Ångströms, an extremely thin graphite target is used. The graphite crystals are sufficiently wide that it is possible for you to find a place where the electron beam is striking a single crystal.

Procedure:

• Set the tube voltage to 6 kV, and keep the beam current below 10 μA. Search for the graphite pattern. This may be difficult as the graphite targets tend to burn away with use. There isn't much graphite left in these tubes, so treat it with care. Keep the beam current low, and take your measurements quickly. Move the beam off of the graphite as soon as your measurements are done. Do not leave the electron beam on the graphite any longer than necessary.

• The graphite pattern will resemble the pattern shown in Fig. 13. If you see rings made of little spots, you're hitting more than one graphite crystal. Move the beam slightly. Once you have a single-crystal pattern, record the diameter (3 measurements) and order of each concentric hexagonal pattern, taking your measurements from the points of the hexagons. In deference to the fragile targets, taking the measurements for only one accelerating voltage is enough.
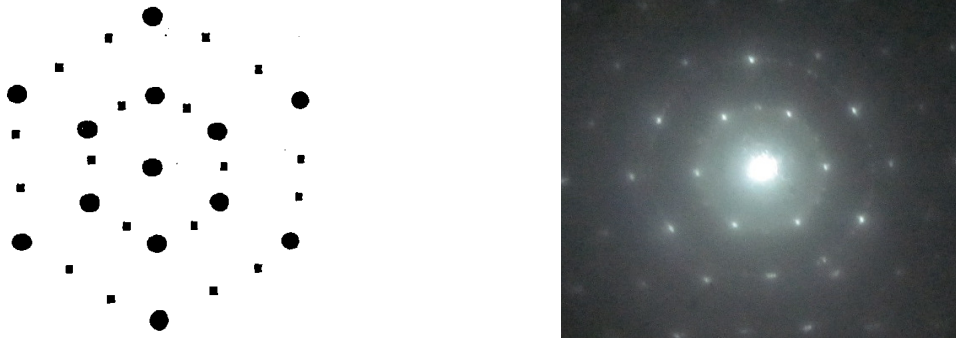
Fig. 13 Diffraction pattern of graphite

IMPORTANT: When you are finished with the apparatus, turn the high voltage down before turning it off. This allows the high voltage capacitors in the power supply to discharge, thus avoiding damage to the tube and power supply.

Data Analysis:

The diffraction pattern for pyrolytic graphite is hexagonal, but rotated 30° with respect to the original lattice. The innermost hexagon corresponds to $n=1$, the next to $n=2$, etc. The lattice constant can be found from Eq. 2 where $d = a \cos 30°$, so that

$$a = \frac{n\lambda L}{r \cos 30°}. \tag{9}$$

Here r is the distance from the center to a point of the hexagon. Using Eq. 9, calculate the value of the lattice constant for each order ($n=1$, 2, 3, etc.) of diffraction. Compare your value with the accepted value of $a = 2.4612$ Å. Estimate your errors in the same manner as in Part 1. Collect your data in a table and discuss your findings.

Final Summary:

In the "conclusions" section of your lab report, summarize your results. Some suggestions: Sketch the diffraction patterns for aluminum and graphite. State which diffraction rings were observed. State your best values for the lattice constants, and compare them with the actual values. Elaborate on the physics that you observed. Write down your overall impressions of the experiment, and how you would change it to improve your results.

References:

Kittel, *Introduction to Solid State Physics*, any edition, Wiley.

Meiners, Eppenstein, Oliva, and Shannon, *Laboratory Physics*, 2nd ed., Wiley, 1986

Taylor and Zafiratos, *Modern Physics for Scientists and Engineers*, Prentice-Hall, 1991. Pay particular attention to Fig. 7.3 (b) on page 153.

Weidner and Sells, *Elementary Modern Physics*, Sec. 5-1 through 5-5. Especially note Fig. 5-7 for comparison with your data.

Nuclear Counting and Statistics

In this lab you will be using a Geiger-Müller counter to measure some of the characteristics of particles being emitted from various radioactive sources.

Before we start the lab we have some SAFETY NOTES: Radioactivity is generally bad for you, especially if you ingest radioactive materials. Although the radioactive sources that you will be handling in our lab are very small, take some basic safety precautions. ABSOLUTELY NO EATING OR DRINKING IN THE LAB. HANDLE THE SOURCES BY THEIR EDGES. (The radioactive material is in a depression in the center of the plastic button). WASH YOUR HANDS BEFORE YOU LEAVE.

We also have an APPARATUS NOTE: NEVER TOUCH THE WINDOW AT THE END OF THE G-M TUBE. The window is very thin and fragile and will break easily.

Background information: Radioactive sources

All of the chemical elements in nature have one or more radioactive isotopes. They are atoms with unstable nuclei. The three major kinds of radioactivity are alpha, beta, and gamma radiation.

Alpha radiation: An alpha particle is made of two protons and two neutrons and is therefore a nucleus of a helium atom. A typical α-decay reaction is $^{210}_{84}\text{Po} \rightarrow ^{206}_{82}\text{Pb} + ^{4}_{2}\text{He} + 5.407\,\text{MeV}$. The 5.407 MeV is seen as the kinetic energy of the decay products, including the alpha particle and any gammas (photons) that may also have been emitted. Note that the total number of protons and neutrons is conserved in this reaction.

Beta radiation: A neutron in the nucleus decays into a proton, an electron, and an anti-neutrino. The proton remains in the nucleus. The electron leaves at great velocity. For historical reasons, the electron is called a "beta" particle. The anti-neutrino also leaves the nucleus. A typical β-decay is $^{90}_{38}\text{Sr} \rightarrow ^{90}_{39}\text{Y} + \beta^{-} + \overline{\nu} + 0.546\,\text{MeV}$. Note that the value of Z has increased by one, but that the number of nucleons has not changed.

Gamma radiation: Gamma rays are high-energy photons. When a radioactive nucleus decays, the "daughter" nucleus may be left in a nuclear excited state. The gammas (high energy photons) come from the decays of these excited states. This is similar to the way that a visible photon is produced from the decay of the excited state of an electron orbiting an atom.

For these experiments, you will use either a ^{137}Cs source that has a gamma energy of 1.17 MeV or a ^{60}Co source which gives off gamma rays at energies of 1.17 MeV and 1.33 MeV as it undergoes beta decay.

Background information: Statistical nature of radioactivity

No one can predict just when a particular nucleus will decay, but when a large number of nuclei are present, the radioactive *decay rate* (number of decays in a unit time) can be measured. When you repeat the measurement, however, you will get slightly different answers because of the inherent randomness of the decays. The actually measured decay rate of a radioactive sample in one trial, let it be *n*, will be around its mean value μ. The probability of measuring *n* decay events in a trial is given by

$$P(n) = \frac{\mu^n}{n!} e^{-\mu}. \tag{1}$$

This probability distribution is called the *Poisson distribution*. The derivation of eq. 1 can be found in most textbook on probability and statistics (e.g., "*Mathematical Methods for Physicists*", by George B. Arfken and Hans J. Weber, 6th edition).

Question 1: Please prove that the Poisson distribution is normalized ($\sum_{n=0}^{\infty} P(n) = 1$) and the mean value of *n* is μ (i.e., $\sum_{n=0}^{\infty} nP(n) = \mu$).

The *standard deviation* of a distribution, often symbolized by σ, is used to represent the degree of randomness of a measurement about its mean value μ. It is defined as

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(n_i - \mu)^2}{N}} = \sqrt{\frac{\sum_{i=1}^{N}(n_i^2 - \mu^2)}{N}}. \tag{2}$$

Here *N* is the total number of measurement, and n_i is the result of the *i*th measurement. The standard deviation shows how wide a distribution is about its mean value. For the Poisson distribution, when *N* is large, the standard deviation is given by

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(n_i^2 - \mu^2)}{N}} = \sqrt{\sum_{n=0}^{\infty} n^2 P(n) - \mu^2} = \sqrt{\mu(\mu+1) - \mu^2} = \sqrt{\mu}. \tag{3}$$

That is, the standard deviation of the Poisson distribution is the square root of the mean value.

Question 2: Please prove Eq. 3.

If the mean value is large (typically $\mu > 20$), the Poisson distribution can be approximated by a Gaussian distribution:

$$P(n) = \frac{\mu^n}{n!} e^{-\mu} \approx \frac{1}{\sqrt{2\pi\mu}} \exp\left[-\frac{(n-\mu)^2}{2\mu}\right]. \tag{4}$$

The proof again can be found in most textbook on probability and statistics. This Gaussian distribution is clearly around its mean value μ, with a standard deviation $\sigma = \sqrt{\mu}$. Like in the

Poisson distribution, the standard deviation of this Gaussian distribution is the square root of the mean value. It can be shown that for this Gaussian distribution 68.3% of the measurement will fall into the region $\mu \pm \sigma = \mu \pm \sqrt{\mu}$. Fig. 1 shows that when the mean value is large, a Poisson distribution evolves into a Gaussian distribution.
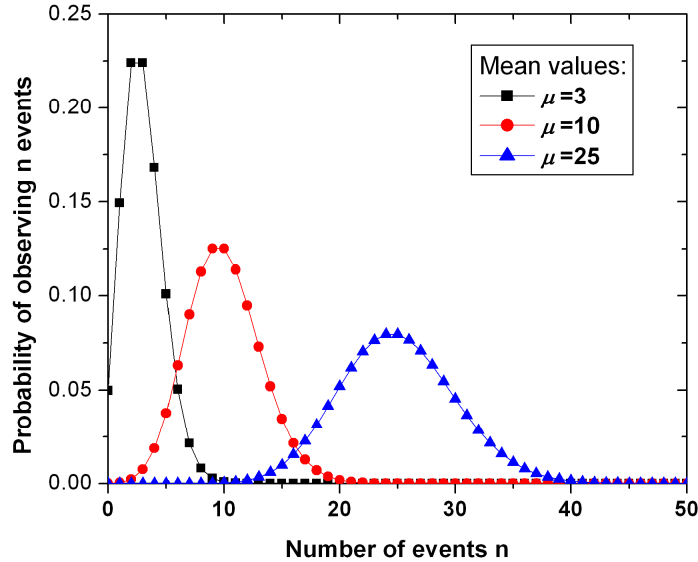

Fig. 1 Poisson distribution with mean values μ=3, 10, and 25.

Because of the randomness of the radioactive decay, we must also measure the error in the decay rate as well as the decay rate itself. The standard deviation is often used to indicate the error of the measurement. Another quantity used for this purpose is the *standard error*, or percent uncertainty, of the measurement, which is defined as

$$\text{standard error} = \frac{\text{standard deviation}}{\text{mean}} \times 100\%. \tag{5}$$

For a Poisson distribution, the standard error is thus $1/\sqrt{\mu}$. This means that the larger the mean value of the measurement, the smaller the standard error, and the more sure you will be on the result of a measurement.

Background information: Radioactive half-life

Although we cannot tell when a particular nucleus will decay, if we have a large number of nuclei we can confidently predict what percentage will decay. The number of particles remaining after some given time is described by an exponential decay curve:

$$N = N_0 \left(\frac{1}{2}\right)^{t/\tau} = N_0 \exp\left(-\ln 2 \times \frac{t}{\tau}\right) \tag{6}$$

8-3

Here N is the number of nuclei left, N_0 the initial number of nuclei, and t the elapsed time. The radioactive half-life, which is the time in which half of the nuclei will decay, is given by τ. Depending on the isotope, half-lives range from less than a microsecond to billions of years.

Question 3: Say that you start with one million particles, and half of them decay every second. Make a graph of N vs. t. Hint: Don't bother with eq. 6. Just repeatedly divide N by 2 and plot the numbers. The resulting curve will be an exponential decay curve. How long does it take to run out of particles?

Background information: The Geiger-Müller counter

A Geiger-Müller counter is a cylindrical capacitor filled with gas. The outer conducting casing of the tube is the cathode, and the anode is a wire in the center of the tube. When a radioactive particle enters the tube, it ionizes some of the gas. The electrons from the gas are accelerated towards the anode, which is held at a positive high voltage. On their way to the anode, the free electrons collide with and ionize other gas atoms. The "secondary electrons" thus freed are also accelerated. Pretty soon there are whole "avalanches" of electrons traveling towards the anode. Sometimes gas atoms are excited by collisions with electrons. The excited atoms emit ultraviolet photons which can also start new "avalanches" of electrons anywhere in the tube. All of the electron avalanches are collected on the anode and constitute an electronic pulse. Each pulse corresponds to one particle crossing through the counter tube. The pulses are counted by an electronic "scaler".

Geiger counters can only register the presence or absence of a charged particle or gamma ray. They are not sensitive to the particle's position, energy, or charge, and they do not detect some neutral particles (particularly neutrons) very well. The gas in a Geiger counter is typically helium or argon with a small percentage of a (usually organic) quenching gas such as ethyl alcohol vapor. The quenching gas limits the duration and size of the avalanches and prevents a massive electrical discharge from occurring in the tube. Discharges can happen anyway if the voltage is too high or if the amount of radiation entering the tube is too large. If a discharge occurs, the Geiger tube won't detect radiation and may be permanently damaged. (This isn't good if it makes you think you're safe when you're really in a high radiation area.) When operated properly and within their limits, Geiger counters are very reliable.

Investigation 1: Statistical variation in the number of counts

We will observe the random nature of radioactivity by repeating the same measurement several hundred times and performing a statistical analysis. We will also investigate the properties of the Poisson distribution and the Gaussian distribution that we have learned above, especially the relation between the mean value and the standard deviation. We will use the small Geiger counter that is interfaced to the computer. The computer will do the drudge work of taking and plotting hundreds of identical measurements.

Procedure:

1. Put the gamma source in a holder and put the holder on the top slot of the wooden box holding the Geiger counter. IMPORTANT: The little "well" on the back of the button contains the radioactive material. This well must face the detector. Turn on the Geiger counter (leaving the sound off, please!)

2. Make sure that the Geiger counter is connected to the LabPro interface. Start the "Logger Pro" software on the computer. Open the file named "Experiments/Nuclear Radiation w Computers/04_Statistics.cmbl". Set up the screen so that you have two graphs. The top graph should be a scatter plot for the measured counts vs. time, and the bottom graph should be a histogram (bar graph) showing the statistical distribution of the data measured so far. The data of the graphs are shown on a data sheet on the left of the window.

3. Set the sampling rate to 1 second/sample in the "Experiment→Data Collection" window, as shown in Fig. 2. This window can also be called out by clicking the clock icon. Set the total time length to 100 seconds, so that we will have 100 sample data collected. Click "Collect" (the green button) to take data. You should have more than 20 counts/second for the mean value of the data. If not please let the instructor know. You can optimize the ranges of the x and y axes of the two graphs when the data are being collected.
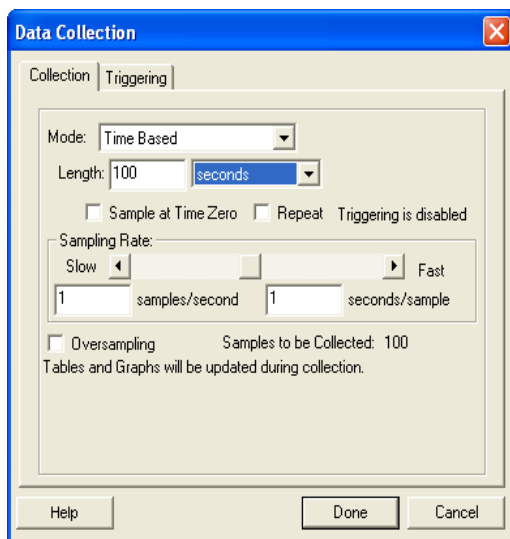

Fig. 2 Data Collection window

4. When the measurement is finished please study a little on the histogram. The "center of gravity" of the curve is the mean value, and one half of the FWHM (full-width at half-maximum) of the curve is roughly the standard deviation. Please estimate these two values. Now activate the counts vs. time graph by clicking on it, and perform a statistics on its data by clicking the "STAT" icon. Move the result window of the statistics so that it does not cover any data points. Are the mean value and the standard deviation match your estimation? Do they roughly satisfy the theoretical relation we have derived (i.e., $\sigma = \sqrt{\mu}$.)? Please note that due to the statistical nature

of the decay, this relation may not be perfectly matched in a measurement, especially when you have limited number of data.

5. Print out the graphs, one copy for each person. It is always a good habit to check the "Print Preview" before you print. You may need to reset the "Page Setup" or make other changes before you actually print. Tape the graphs in your lab notebook. In your lab report, verbally describe what you are seeing and what the graphs mean. Please record the mean value, the standard deviation, and the standard error in Table 1 shown below.

6. Set the sampling rate to 2 second/sample. Set the total time length to 200 seconds, so that we will again have 100 sample data collected. Repeat steps 4 and 5. Compare the standard errors for the 2-second/sample and the 1-second/sample runs. Does taking a larger data number by using a longer time interval for each data reduce the standard error?

7. Set the data collection time interval to 4 second/sample. Set the total time length to 400 seconds, and repeat step 6. Please record all the statistical results and finish Table 1.

Table 1

Time interval/sample	Mean (from the statistical result)	Standard deviation (from the statistical result)	$\sqrt{\text{Mean}}$	Standard error $\left(=\dfrac{\text{standard deviation}}{\text{mean}}\right)$
1s/sample				
2s/sample				
4s/sample				

Question 4: Using your apparatus, how long would you have to count for one data in order to get a 1% standard error?

Physics note: The randomness in this experiment is a property of nature. The variations that you are seeing in the number of counts are real. The differences in the number of counts from one measurement to another are NOT experimental error. They are due to the fact that the decays are controlled by quantum mechanics, which can only describe nature statistically.

Note: If you want to know more about your results in this investigation, you'll find the information listed under "Poisson statistics" in the index of almost any mathematics book on probability and statistics. Poisson statistics is also discussed in books on nuclear counting and instrumentation.

Investigation 2: Measuring the half-life of a radioactive sample

When ^{137}Cs β–decays, the daughter nucleus is ^{137}Ba, which has a half-life of several minutes. You will measure this half-life. The instructor will prepare a sample for you by washing some ^{137}Ba out of a ^{137}Cs sample with dilute HCl (hydrochloric acid).

Procedure: (READ THIS FIRST! You must work FAST after you start.)

1. Set up the computer to take a reading at 10-second intervals. (You may have to play with this number a bit.) The computer must continue to take readings for a total time of about 10 minutes. Turn off the histogram feature; you won't need it here. Start the computer and make sure that it's taking the readings properly.

2. When you are sure that you are ready to go, ask the instructor for a sample. Start taking data the instant that you get your sample. Time is of the essence. Once you start, you cannot stop until the data run is finished.

3. Estimate the half-life of your sample as follows: Pick up a point on your curve (not necessarily a real data point) and read off the number of counts. Find a corresponding point with half the number of counts as your first point on your curve. The half-life is the elapsed time between these two points.

4. Use the curve fit capability of the computer to find the half-life. This is done by clicking on the "f(x)=" icon, and choose the "Natural Exponent" function. Look at the parameters that the computer used in the curve fit. Use these parameters to obtain the half-life of the source. How does this value fit compare to the one you did in step 3? Print out the graph of your data and the fitted curve.

Question 5: Look up the actual half-life of ^{137}Ba and compare it with your measured value. Is the accepted value within your error?

In your lab report please summarize what you have learned in today's lab.

Graphical Schrödinger Equation

Objectives of this lab:

- Explore the solutions to the one-dimensional Schrödinger Equation (SE) for some simple potential energy functions.

- Observe the relationship between wavelength, energy and potential energy for a quantum mechanical system.

- Observe the manner in which the wave function changes as the potential changes suddenly at steps or barriers.

- Compare the solutions of the SE in the finite well to those of the 1-D box (infinite well).

- Calculate the coefficients of the reflected and transmitted waves in the case of an incoming wave encountering a "step up" in the potential energy.

Description of the program:

The program, *Graphical Schrödinger Equation*, plots solutions of the Schrödinger equation

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}[E - U(x)]\psi \tag{1}$$

for a series of different potential energy curves and for several different scenarios. The potential energy curves are classified in the following way.

Steps and Barriers: First of all, there are the potential steps and barriers which give rise to the scattering of plane waves (waves with a single wavelength). Potential steps involve a discrete, finite change in the otherwise constant potential energy at the location of the step. Potential barriers are made from two steps, one a step up and the other a step down. The two steps produce a region of higher (constant) potential energy, which can act like a barrier to moving particles. The behavior of the wave function in the regions of different potential energy depends on the energy, *E*, and mass, *m*, of the particle. In particular, those two parameters determine the wavenumber, *k*, of the wave function:

$$k = \sqrt{\frac{2m}{\hbar^2}[E - U(x)]} \tag{2}$$

The greater the mass of the particle, for a given energy, the greater the value of *k* and the smaller the wavelength, $\lambda = 2\pi/k$. The greater the difference between the particle energy and the potential energy in that region, the greater the value of *k*, and the smaller the wavelength.

Solutions: The solution to the Schrödinger equation in a region of a constant potential, U_0, such that $E-U_0$ is positive, is given by:

$$\psi(x) = Ae^{ikx} + Be^{-ikx}. \tag{3}$$

The first term represents a particle or stream of particles passing to the right, while the second term represents a particle or stream of particles passing to the left. Both types of motion must be allowed for, unless there is a physical reason to exclude one or the other. For example, in the case of a stream of particles coming from the left, moving to the right and encountering a potential step at $x = 0$, the left-moving stream is associated with reflection of the incoming stream at the sudden change in potential. In the region for $x > 0$, there should not be any left-moving stream, since past that position there are no other obstacles for producing reflections.

If part of the solution domain has a potential energy, $U_0 > E$, then the Schrödinger equation becomes

$$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(U_0 - E)\psi. \tag{4}$$

If we define

$$\kappa = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} \tag{5}$$

so that κ is real, then the wave function becomes in this case

$$\psi(x) = Ae^{\kappa x} + Be^{-\kappa x}. \tag{6}$$

The solutions are now real exponentials, which do not oscillate, but either grow or decay with x. If this region extends to positive infinity, then the exponentially growing term (the first one in the expression for ψ) must have a zero coefficient. Otherwise, the wave function would become infinite at positive infinity, and all of the probability would be there.

Matching conditions: The position at which the potential suddenly changes has to match the solutions obtained for the different potentials on either side. As we have discussed in class, the wave function and its first derivatives must be continuous across that position. That is, for a sudden change in the potential at $x = a$,

$$\psi_I(a) = \psi_{II}(a),$$
$$\left.\frac{d\psi_I}{dx}\right|_{x=a} = \left.\frac{d\psi_{II}}{dx}\right|_{x=a}. \tag{7}$$

If the coefficient of the left-most incoming wave function is assumed to be fixed by the experiment (that is, the experimenter sets the amplitude of the incoming particle wave), and if there is no left-moving wave in the right-most region in the problem domain, then these conditions imposed at all of the positions where the potential changes are enough to solve for all of the coefficients in Equations 3 and 6 in terms of the known incoming amplitude.

Solving Problems:

I) Reflection and transmission at a potential step. In this scenario, a wave is traveling to the right, and encounters a step up in the potential at $x = 0$, from $U = 0$ to $U = U_0$. Assume that the particle energy E is twice of U_0, that is, $E = 2U_0$. Determine the wave function ψ for this potential, assuming that the amplitude of the incoming wave is just $A = 1$.

- First, draw a sketch of the potential energy function. Indicate the position where the potential changes suddenly. Also, use a dashed line to indicate the particle energy relative to the potential.

- Second, write down the most general form of the wavefunction in each region, taking into account any assumptions, and identify the type of wave each part of each wavefunction is associated with.

- Third, write down the value of the wavenumber for each of the regions, as in Equations 2 and 5, in terms of the energy and potential. How much bigger is the wavelength in the region of potential U_0 compared to the region where the potential is 0?

- Now, apply the matching conditions to solve for the coefficients of your wave function. Remember that $A = 1$ for the right-moving wave in the left-most region. Your coefficients should be expressed in terms of the wavenumbers from both regions.

- Finally, use the GSE program to set up a scenario like this by clicking on the Barrier Potentials menu button, and choosing Step. You are free to choose the particle energy, mass and step height. By clicking on the Parameters menu button, you can adjust the particle energy and step height to satisfy the requirements of this problem. Note the values that you use. By clicking on the wave function display menu item, you can choose whether to view the real or imaginary part of the wavefunction, and whether to view the total wavefunction, or just the scattered or incident part. For the step potential problem, you can only view the total wave. Choose to view the real part.

- Now, display the solution by clicking on the Solve It menu button. Look at your solution. Compare the wavelengths in the two regions. Do they agree with what you predicted above? Make a sketch of the wave. Did the matching conditions do what they were supposed to do?

- You can print your picture by pressing the "Alt+PrintScreen" keys, and then paste the memory into PowerPoint or Paint to print. You can also use the *snipping tool* provided by Windows to copy the picture.

II) Burrowing into a step: Now set up a scenario in which the particle energy is only half that of the potential past the step. Solve the problem as in the first case, and plot it on GSE. Note from your solution the value of the coefficient for the wavefunction in the region past the step, and also note the plot on GSE. What does that tell you about the probability of the particle penetrating some distance into the barrier?

III) Tunneling through a barrier: Now, go back to the main menu, and chose the barrier potential in the Potential Barriers menu item. Set up your energy to be half that of the potential

barrier height, and notice the width of the barrier, relative to the wavelength of your incoming wave. Solve the SE, and look at the scattered, incident and total waves. What does the amplitude of the scattered wave solution in particular tell you about the probability of the wave making it through the barrier?

Adjust the barrier width. How wide does the barrier need to be, relative to the wavelength, for the transmission probability to become nearly zero?

IV) Square wells: In the Well Potential menu, chose the Square Well. Solve the problem and let the computer display the energies and the wave functions. Examine the relation between the wavelength and the energy. Observe the behavior of the wave function at the boundaries of the well. Change the depth and width of the well, and watch how the energy levels and the wave functions change.

V) Harmonic Oscillators: In the Well Potential menu, chose the Harmonic Oscillator. Solve the problem and let the computer display the energies and the wave functions. Examine the separation between the energy levels. Examine the relation between the wavelength and the energy. Observe the behavior of the wave function at the boundaries of the well. Change the width of the well, and watch how the energy levels and the wave functions change. Observe the wave distribution at low and high energy levels. Does it agree with classical physics?

Appendices: Math helps on solving the Schrödinger equations

1) Reflection and transmission at a potential step.

E

U_0
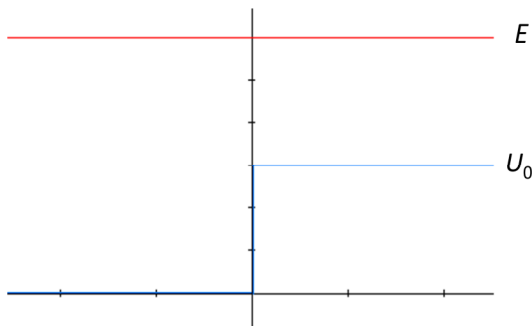
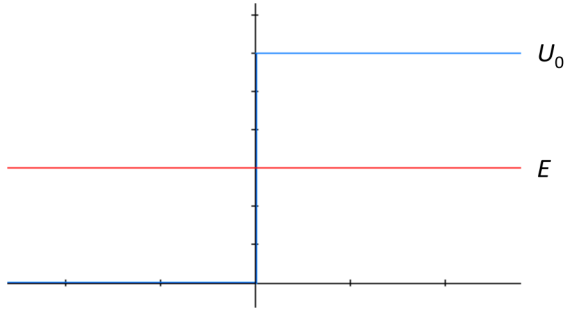$$\psi_I(x) = e^{ikx} + re^{-ikx}, \quad k = \sqrt{\frac{2mE}{\hbar^2}}.$$

$$\psi_{II}(x) = te^{ik'x}, \quad k' = \sqrt{\frac{2m(E-U_0)}{\hbar^2}}.$$

$$\left.\begin{array}{l} \psi_I(0) = \psi_{II}(0) \Rightarrow 1 + r = t \\[2mm] \dfrac{d\psi_I}{dx}\bigg|_{x=0} = \dfrac{d\psi_{II}}{dx}\bigg|_{x=0} \Rightarrow ik(1-r) = ik't \end{array}\right\} \Rightarrow r = \frac{k-k'}{k+k'}.$$

$$R = |r|^2 = \left(\frac{\sqrt{E} - \sqrt{E-U_0}}{\sqrt{E} + \sqrt{E-U_0}}\right)^2 = \frac{U_0^2}{\left(\sqrt{E} + \sqrt{E-U_0}\right)^4}.$$

For $E = 2U_0$, $R = 0.029$.

2) Burrowing into a step.

$$\psi_I(x) = e^{ikx} + re^{-ikx}. \quad k = \sqrt{\frac{2mE}{\hbar^2}}.$$

$$\psi_{II}(x) = Ae^{\kappa x} + Be^{-\kappa x} = Be^{-\kappa x}. \quad \kappa = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}}.$$

$$\left.\begin{array}{l} \psi_I(0) = \psi_{II}(0) \Rightarrow 1 + r = B \\[2mm] \dfrac{d\psi_I}{dx}\bigg|_{x=0} = \dfrac{d\psi_{II}}{dx}\bigg|_{x=0} \Rightarrow ik(1-r) = -\kappa B \end{array}\right\} \Rightarrow r = \frac{ik + \kappa}{ik - \kappa}.$$

$$|R| = 1.$$