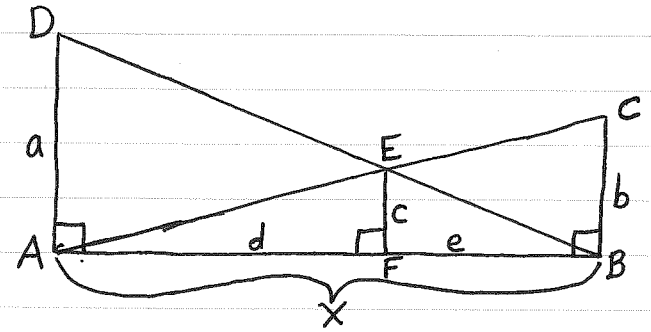
$HM(a,b) = 2c$

$c = \frac{ab}{a+b}$



$\triangle DAB \sim \triangle EFB$

$\triangle CBA \sim \triangle EFA$

$\frac{a}{c} = \frac{x}{e} \Rightarrow a = \frac{xc}{e}$

$\frac{b}{c} = \frac{x}{d} \Rightarrow b = \frac{xc}{d}$

$a + b = \frac{xc}{e} + \frac{xc}{d}$

$= \frac{dxc + exc}{ed} = \frac{xc(e+d)}{ed}$

$ab = \frac{xc}{e} * \frac{xc}{d}$

$= \frac{xc * xc}{ed}$

$\frac{ab}{a+b} = \frac{xc * xc}{ed} \div \frac{xc(e+d)}{ed}$

$= \frac{xc * \cancel{xc}}{\cancel{ed}} * \frac{\cancel{ed}}{(e+d)\cancel{xc}}$

$= \frac{xc}{e+d} = \frac{\cancel{x}c}{\cancel{x}} = c = \frac{ab}{a+b}$