

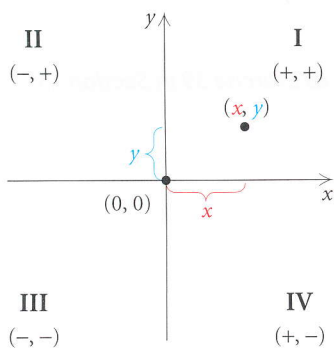
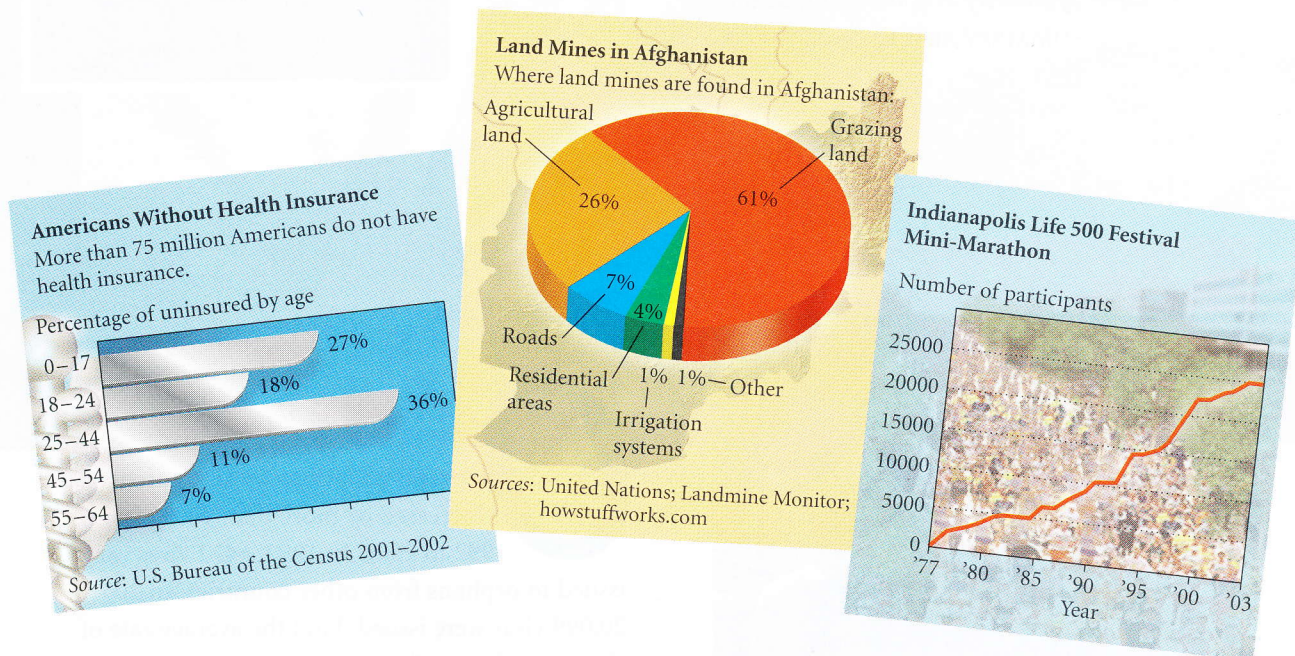
1.1

Introduction to Graphing

- Plot points.
- Determine whether an ordered pair is a solution of an equation.
- Graph equations.
- Find the distance between two points in the plane and find the midpoint of a segment.
- Find an equation of a circle with a given center and radius, and given an equation of a circle, find the center and the radius.
- Graph equations of circles.

Graphs

Graphs provide a means of displaying, interpreting, and analyzing data in a visual format. It is not uncommon to open a newspaper or magazine and encounter graphs. Examples of bar, circle, and line graphs are shown below.

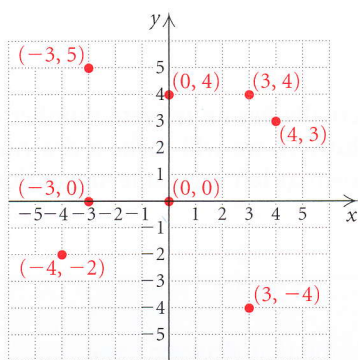


Many real-world situations can be modeled, or described mathematically, using equations in which two variables appear. We use a plane to graph a pair of numbers. To locate points on a plane, we use two perpendicular number lines, called **axes**, which intersect at $(0, 0)$. We call this point the **origin**. The horizontal axis is called the **x-axis**, and the vertical axis is called the **y-axis**. (Other variables, such as a and b , can also be used.) The axes divide the plane into four regions, called **quadrants**, denoted by Roman numerals and numbered counterclockwise from the upper right. Arrows show the positive direction of each axis.

Each point (x, y) in the plane is described by an **ordered pair**. The first number, x , indicates the point's horizontal location with respect to the y -axis, and the second number, y , indicates the point's vertical location with

respect to the x -axis. We call x the **first coordinate**, **x -coordinate**, or **abscissa**. We call y the **second coordinate**, **y -coordinate**, or **ordinate**. Such a representation is called the **Cartesian coordinate system** in honor of the French mathematician and philosopher René Descartes (1596–1650).

In the first quadrant, both coordinates of a point are positive. In the second quadrant, the first coordinate is negative and the second is positive. In the third quadrant, both coordinates are negative, and in the fourth quadrant, the first coordinate is positive and the second is negative.



EXAMPLE 1 Graph and label the points $(-3, 5)$, $(4, 3)$, $(3, 4)$, $(-4, -2)$, $(3, -4)$, $(0, 4)$, $(-3, 0)$, and $(0, 0)$.

Solution To graph or **plot** $(-3, 5)$, we note that the x -coordinate, -3 , tells us to move from the origin 3 units to the left of the y -axis. Then we move 5 units up from the x -axis.* To graph the other points, we proceed in a similar manner. (See the graph at left.) Note that the point $(4, 3)$ is different from the point $(3, 4)$.

Solutions of Equations

Equations in two variables, like $2x + 3y = 18$, have solutions (x, y) that are ordered pairs such that when the first coordinate is substituted for x and the second coordinate is substituted for y , **the result is a true equation**. The first coordinate in an ordered pair generally represents the variable that occurs first alphabetically.

EXAMPLE 2 Determine whether each ordered pair is a solution of $2x + 3y = 18$.

a) $(-5, 7)$

b) $(3, 4)$

Solution We substitute the ordered pair into the equation and determine whether the resulting equation is true.

a) $2x + 3y = 18$

$$\begin{array}{r|l} 2(-5) + 3(7) & ? 18 \\ -10 + 21 & \\ \hline 11 & 18 \quad \text{FALSE} \end{array} \quad \begin{array}{l} \text{We substitute } -5 \text{ for } x \text{ and} \\ 7 \text{ for } y \text{ (alphabetical order).} \end{array}$$

The equation $11 = 18$ is false, so $(-5, 7)$ is not a solution.

b) $2x + 3y = 18$

$$\begin{array}{r|l} 2(3) + 3(4) & ? 18 \\ 6 + 12 & \\ \hline 18 & 18 \quad \text{TRUE} \end{array} \quad \begin{array}{l} \text{We substitute } 3 \text{ for } x \\ \text{and } 4 \text{ for } y. \end{array}$$

The equation $18 = 18$ is true, so $(3, 4)$ is a solution.

*We first saw notation such as $(-3, 5)$ in Section R.1. There the notation represented an open interval. Here the notation represents an ordered pair. The context in which the notation appears usually makes the meaning clear.

We can also perform these substitutions on a graphing calculator. When we substitute -5 for x and 7 for y , we get 11 . Since $11 \neq 18$, $(-5, 7)$ is not a solution of the equation. When we substitute 3 for x and 4 for y , we get 18 , so $(3, 4)$ is a solution.

$2(-5)+3*7$	11
$2*3+3*4$	18

Graphs of Equations

The equation considered in Example 2 actually has an infinite number of solutions. Since we cannot list all the solutions, we will make a drawing, called a **graph**, that represents them. Shown at left are some suggestions for drawing graphs.

To Graph an Equation

To **graph an equation** is to make a drawing that represents the solutions of that equation.

Graphs of equations of the type $Ax + By = C$ are straight lines. Many such equations can be graphed conveniently using intercepts. The **x -intercept** of the graph of an equation is the point at which the graph crosses the x -axis. The **y -intercept** is the point at which the graph crosses the y -axis. We know from geometry that only one line can be drawn through two given points. Thus, if we know the intercepts, we can graph the line. To ensure that a computational error has not been made, it is a good idea to calculate and plot a third point as a check.

x - and y -Intercepts

An **x -intercept** is a point $(a, 0)$. To find a , let $y = 0$ and solve for x .
A **y -intercept** is a point $(0, b)$. To find b , let $x = 0$ and solve for y .

EXAMPLE 3 Graph: $2x + 3y = 18$.

Solution The graph is a line. To find ordered pairs that are solutions of this equation, we can replace either x or y with any number and then solve for the other variable. In this case, it is convenient to find the intercepts of the graph. For instance, if x is replaced with 0 , then

$$\begin{aligned} 2 \cdot 0 + 3y &= 18 \\ 3y &= 18 \\ y &= 6. \quad \text{Dividing by 3} \end{aligned}$$

Thus, $(0, 6)$ is a solution. It is the y -intercept of the graph. If y is replaced with 0, then

$$2x + 3 \cdot 0 = 18$$

$$2x = 18$$

$$x = 9. \quad \text{Dividing by 2}$$

Thus, $(9, 0)$ is a solution. It is the x -intercept of the graph. We find a third solution as a check. If x is replaced with 5, then

$$2 \cdot 5 + 3y = 18$$

$$10 + 3y = 18$$

$$3y = 8 \quad \text{Subtracting 10}$$

$$y = \frac{8}{3}. \quad \text{Dividing by 3}$$

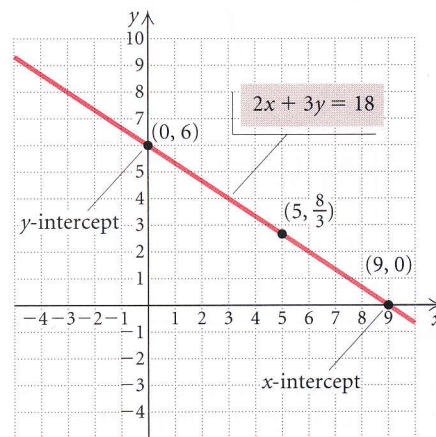
Thus, $(5, \frac{8}{3})$ is a solution.

We list the solutions in a table and then plot the points. Note that the points appear to lie on a straight line.

Suggestions for Drawing Graphs

1. Calculate solutions and list the ordered pairs in a table.
2. Use graph paper.
3. Draw axes and label them with the variables.
4. Use arrows on the axes to indicate positive directions.
5. Scale the axes; that is, label the tick marks on the axes. Consider the ordered pairs found in step (1) when choosing the scale.
6. Plot the ordered pairs, look for patterns, and complete the graph. Label the graph with the equation being graphed.

x	y	(x, y)
0	6	$(0, 6)$
9	0	$(9, 0)$
5	$\frac{8}{3}$	$(5, \frac{8}{3})$



Were we to graph additional solutions of $2x + 3y = 18$, they would be on the same straight line. Thus, to complete the graph, we use a straight-edge to draw a line, as shown in the figure. This line represents all solutions of the equation. Every point on the line represents a solution; every solution is represented by a point on the line.

When graphing some equations, it is easier to first solve for y and then find ordered pairs. We can use the addition and multiplication principles to solve for y .

GCM EXAMPLE 4 Graph: $3x - 5y = -10$.

Solution We first solve for y :

$$3x - 5y = -10$$

$$-5y = -3x - 10 \quad \text{Subtracting } 3x \text{ on both sides}$$

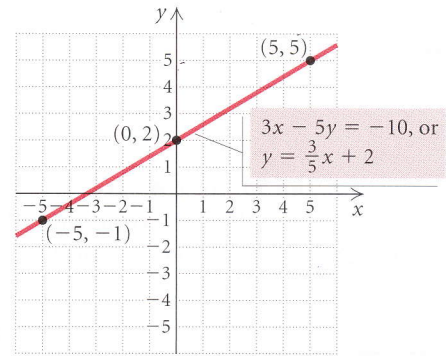
$$y = \frac{3}{5}x + 2. \quad \text{Multiplying by } -\frac{1}{5} \text{ on both sides}$$

By choosing multiples of 5 for x , we can avoid fraction values when calculating y . For example, if we choose -5 for x , we get

$$y = \frac{3}{5}x + 2 = \frac{3}{5}(-5) + 2 = -3 + 2 = -1.$$

The following table lists a few more points. We plot the points and draw the graph.

x	y	(x, y)
-5	-1	$(-5, -1)$
0	2	$(0, 2)$
5	5	$(5, 5)$



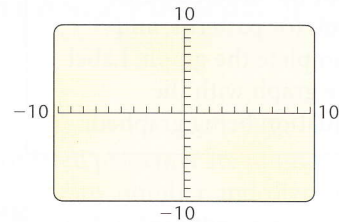
In the equation $y = \frac{3}{5}x + 2$, the value of y depends on the value chosen for x , so x is said to be the **independent variable** and y the **dependent variable**.

We can graph an equation on a graphing calculator. Many calculators require an equation to be entered in the form “ $y =$.” In such a case, if the equation is not initially given in this form, it must be solved for y before it is entered. For the equation $3x - 5y = -10$ in Example 4, we enter $y = \frac{3}{5}x + 2$ on the equation-editor, or “ $y =$,” screen in the form $y = (3/5)x + 2$, as shown in the window at left.

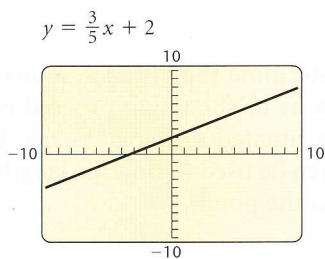
Next, we determine the portion of the xy -plane that will appear on the calculator’s screen. That portion of the plane is called the **viewing window**. The notation used in this text to denote a window setting consists of four numbers $[L, R, B, T]$, which represent the **L**eft and **R**ight endpoints of the x -axis and the **B**ottom and **T**op endpoints of the y -axis, respectively. The window with the settings $[-10, 10, -10, 10]$ is the **standard viewing window**. On some graphing calculators, the standard window can be selected quickly using the ZSTANDARD feature from the ZOOM menu.

```
Plot1 Plot2 Plot3
\Y1=(3/5)X+2
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
WINDOW
Xmin = -10
Xmax = 10
Xscl = 1
Ymin = -10
Ymax = 10
Yscl = 1
Xres = 1
```



X_{\min} and X_{\max} are used to set the left and right endpoints of the x -axis, respectively; Y_{\min} and Y_{\max} are used to set the bottom and top endpoints of the y -axis. The settings X_{scl} and Y_{scl} give the scales for the axes. For example, $X_{\text{scl}} = 1$ and $Y_{\text{scl}} = 1$ means that there is 1 unit between tick marks on each of the axes.

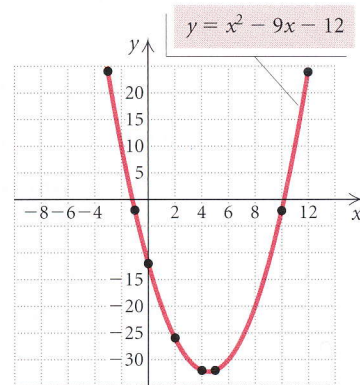


After entering the equation $y = (3/5)x + 2$ and choosing a viewing window, we can then draw the graph shown at left.

GCM EXAMPLE 5 Graph: $y = x^2 - 9x - 12$.

Solution Note that this is not of the form $Ax + By = C$, thus it is not linear. We make a table of values, plot enough points to obtain an idea of the shape of the curve, and connect them with a smooth curve. It is important to scale the axes to include most of the ordered pairs listed in the table. Here it is appropriate to use a larger scale on the y -axis than on the x -axis.

x	y	(x, y)
-3	24	$(-3, 24)$
-1	-2	$(-1, -2)$
0	-12	$(0, -12)$
2	-26	$(2, -26)$
4	-32	$(4, -32)$
5	-32	$(5, -32)$
10	-2	$(10, -2)$
12	24	$(12, 24)$



- ① Select values for x .
- ② Compute values for y .

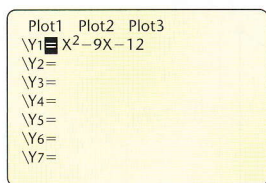


Figure 1

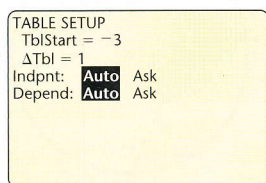


Figure 2

A graphing calculator can be used to create a table of ordered pairs that are solutions of an equation. For the equation in Example 5, $y = x^2 - 9x - 12$, we first enter the equation on the equation-editor screen (see Fig. 1). Then we set up a table in AUTO mode by designating a value for TBLSTART and a value for Δ TBL. The calculator will produce a table starting with the value of TBLSTART and continuing by adding Δ TBL to supply succeeding x -values. For the equation $y = x^2 - 9x - 12$, we let TBLSTART = -3 and Δ TBL = 1 (see Fig. 2).

We can scroll up and down in the table to find values other than those shown in Fig. 3. We can also graph this equation on the graphing calculator, as shown in Fig. 4.

X	Y1
-3	24
-2	10
-1	-2
0	-12
1	-20
2	-26
3	-30

X = -3

Figure 3

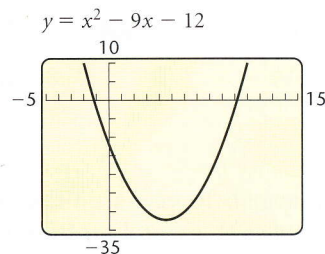
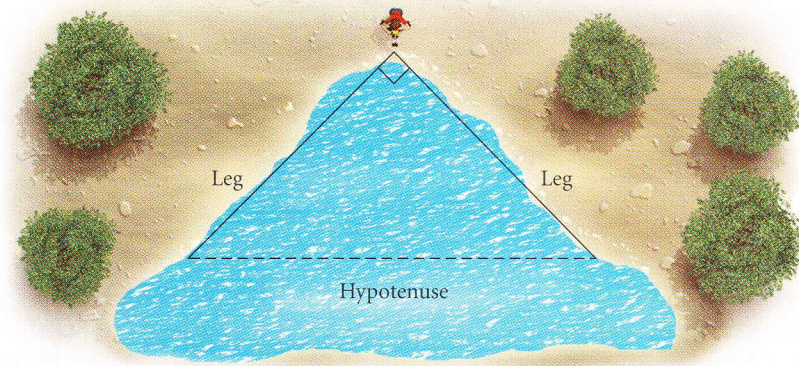


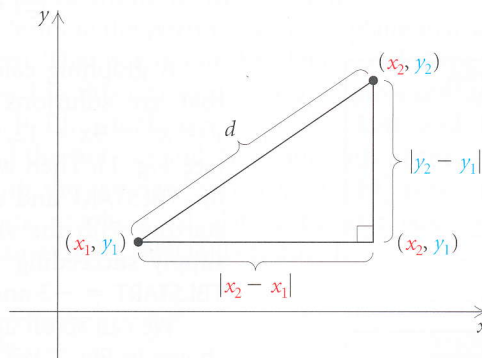
Figure 4

The Distance Formula

Suppose that a conservationist needs to determine the distance across an irregularly shaped pond. One way in which he or she might proceed is to measure two legs of a right triangle that is situated as shown below. The Pythagorean theorem, $a^2 + b^2 = c^2$, can then be used to find the length of the hypotenuse, which is the distance across the pond.



A similar strategy is used to find the distance between two points in a plane. For two points (x_1, y_1) and (x_2, y_2) , we can draw a right triangle in which the legs have lengths $|x_2 - x_1|$ and $|y_2 - y_1|$.



Using the Pythagorean theorem, we have

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2.$$

Because we are squaring, parentheses can replace the absolute-value symbols:

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

Taking the principal square root, we obtain the distance formula.

The Distance Formula

The **distance** d between any two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

The subtraction of the x -coordinates can be done in any order, as can the subtraction of the y -coordinates. Although we derived the distance formula by considering two points not on a horizontal or a vertical line, the distance formula holds for *any* two points.

EXAMPLE 6 Find the distance between the pair of points.

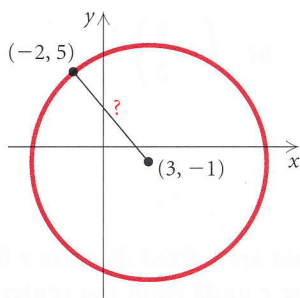
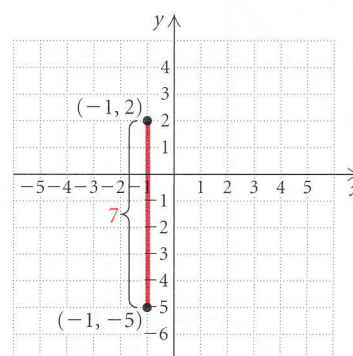
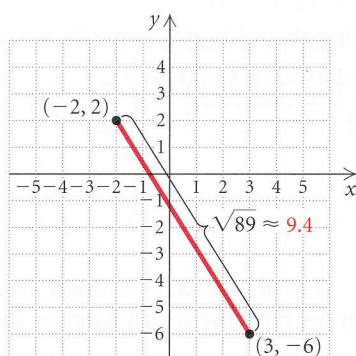
a) $(-2, 2)$ and $(3, -6)$

b) $(-1, -5)$ and $(-1, 2)$

Solution We substitute into the distance formula.

$$\begin{aligned} \text{a) } d &= \sqrt{[3 - (-2)]^2 + (-6 - 2)^2} \\ &= \sqrt{5^2 + (-8)^2} = \sqrt{25 + 64} \\ &= \sqrt{89} \approx 9.4 \end{aligned}$$

$$\begin{aligned} \text{b) } d &= \sqrt{[-1 - (-1)]^2 + (-5 - 2)^2} \\ &= \sqrt{0^2 + (-7)^2} = \sqrt{0 + 49} \\ &= \sqrt{49} = 7 \end{aligned}$$



EXAMPLE 7 The point $(-2, 5)$ is on a circle that has $(3, -1)$ as its center. Find the length of the radius of the circle.

Solution Since the length of the radius is the distance from the center to a point on the circle, we substitute into the distance formula:

$$\begin{aligned} r &= \sqrt{[3 - (-2)]^2 + (-1 - 5)^2} && \text{Either point can serve as } (x_1, y_1). \\ &= \sqrt{5^2 + (-6)^2} = \sqrt{61} \approx 7.8. && \text{Rounded to the nearest tenth} \end{aligned}$$

The radius of the circle is approximately 7.8.

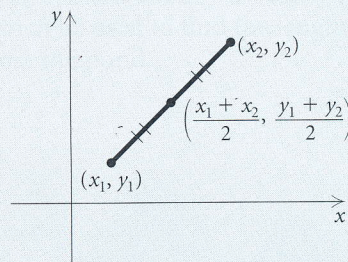
Midpoints of Segments

The distance formula can be used to develop a way of determining the *midpoint* of a segment when the endpoints are known. We state the formula and leave its proof to the exercise set.

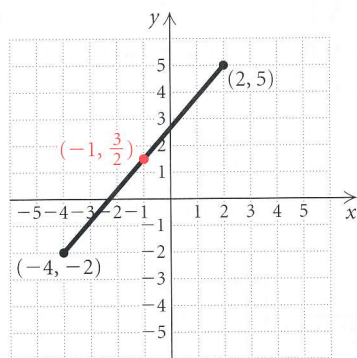
The Midpoint Formula

If the endpoints of a segment are (x_1, y_1) and (x_2, y_2) , then the coordinates of the **midpoint** are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$



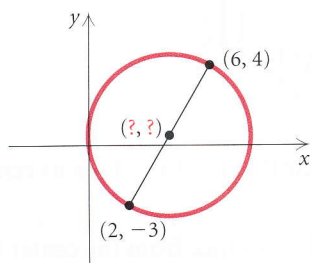
Note that we obtain the coordinates of the midpoint by averaging the coordinates of the endpoints. This is a good way to remember the midpoint formula.



EXAMPLE 8 Find the midpoint of the segment whose endpoints are $(-4, -2)$ and $(2, 5)$.

Solution Using the midpoint formula, we obtain

$$\begin{aligned} \left(\frac{-4 + 2}{2}, \frac{-2 + 5}{2} \right) &= \left(\frac{-2}{2}, \frac{3}{2} \right) \\ &= \left(-1, \frac{3}{2} \right). \end{aligned}$$

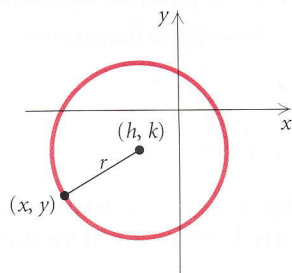


EXAMPLE 9 The diameter of a circle connects two points $(2, -3)$ and $(6, 4)$ on the circle. Find the coordinates of the center of the circle.

Solution Since the center of the circle is the midpoint of the diameter, we use the midpoint formula:

$$\left(\frac{2 + 6}{2}, \frac{-3 + 4}{2} \right), \text{ or } \left(\frac{8}{2}, \frac{1}{2} \right), \text{ or } \left(4, \frac{1}{2} \right).$$

The coordinates of the center are $(4, \frac{1}{2})$.



Circles

A **circle** is the set of all points in a plane that are a fixed distance r from a **center** (h, k) . Thus if a point (x, y) is to be r units from the center, we must have

$$r = \sqrt{(x - h)^2 + (y - k)^2}. \quad \text{Using the distance formula}$$

Squaring both sides gives an equation of a circle. The distance r is the length of a **radius** of the circle.

The Equation of a Circle

The equation of a circle with center (h, k) and radius r , in standard form, is

$$(x - h)^2 + (y - k)^2 = r^2.$$

EXAMPLE 10 Find an equation of the circle having radius 5 and center $(3, -7)$.

Solution Using the standard form, we have

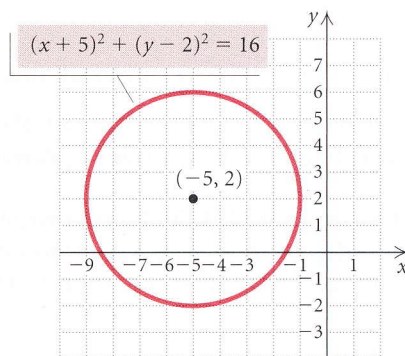
$$\begin{aligned} [x - 3]^2 + [y - (-7)]^2 &= 5^2 && \text{Substituting} \\ (x - 3)^2 + (y + 7)^2 &= 25. \end{aligned}$$

GCM EXAMPLE 11 Graph the circle $(x + 5)^2 + (y - 2)^2 = 16$.

Solution We write the equation in standard form to determine the center and the radius:

$$[x - (-5)]^2 + [y - 2]^2 = 4^2.$$

The center is $(-5, 2)$ and the radius is 4. We locate the center and draw the circle using a compass.



If we square $x + 5$ and $y - 2$ in the equation of the circle in Example 11, we obtain another form for the equation:

$$\begin{aligned} (x + 5)^2 + (y - 2)^2 &= 16 \\ x^2 + 10x + 25 + y^2 - 4y + 4 &= 16 && \text{Squaring} \\ x^2 + y^2 + 10x - 4y + 13 &= 0. && \text{Simplifying} \end{aligned}$$

This form is the general form of the equation of the circle. With equations in general form, we can complete the square to find the center and the radius of the circle.

EXAMPLE 12 Find the center and the radius of the circle and graph it:

$$x^2 + y^2 - 8x + 2y + 13 = 0.$$

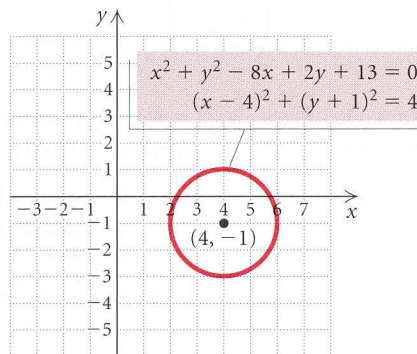
Solution First, we regroup the terms in order to **complete the square** twice, once with $x^2 - 8x$ and once with $y^2 + 2y$:

$$\begin{aligned} x^2 + y^2 - 8x + 2y + 13 &= 0 \\ (x^2 - 8x) + (y^2 + 2y) + 13 &= 0. \quad \text{Regrouping} \end{aligned}$$

Next, we complete the square inside each set of parentheses. We want to add something to both $x^2 - 8x$ and $y^2 + 2y$ so that each becomes the square of a binomial. For $x^2 - 8x$, we take half the x -coefficient, $\frac{1}{2}(-8) = -4$, and square it, $(-4)^2 = 16$. Then we add 0, or $16 - 16$, inside the parentheses. For $y^2 + 2y$, we have $\frac{1}{2} \cdot 2 = 1$ and $1^2 = 1$, so we add $1 - 1$ inside the parentheses.

$$\begin{aligned} (x^2 - 8x + 0) + (y^2 + 2y + 0) + 13 &= 0 && \text{Adding 0} \\ (x^2 - 8x + 16 - 16) + (y^2 + 2y + 1 - 1) + 13 &= 0 && \text{Regrouping} \\ (x^2 - 8x + 16) + (y^2 + 2y + 1) - 16 - 1 + 13 &= 0 && \text{Factoring and} \\ &&& \text{simplifying} \\ (x - 4)^2 + (y + 1)^2 - 4 &= 0 && \text{Adding 4} \\ (x - 4)^2 + (y + 1)^2 &= 4 && \text{Writing in} \\ (x - 4)^2 + [y - (-1)]^2 &= 2^2 && \text{standard form} \end{aligned}$$

The center is $(4, -1)$ and the radius is 2.



Circles can also be graphed using a graphing calculator. We show one method of doing so here. Another method is discussed in Section 6.2.

When we graph a circle, we select a viewing window in which the distance between units is visually the same on both axes. This procedure is called **squaring the viewing window**. We do this so that the graph will not be distorted. A graph of the circle $x^2 + y^2 = 36$ in a nonsquared window is shown in Fig. 1.

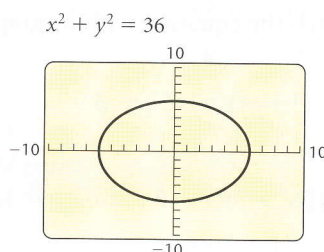


Figure 1

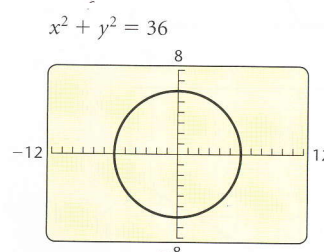


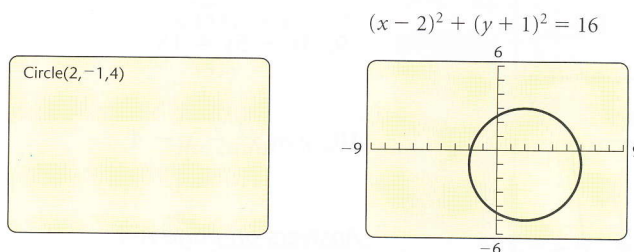
Figure 2

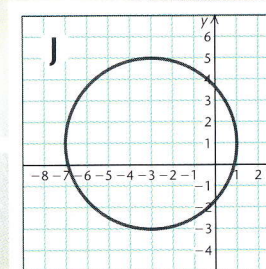
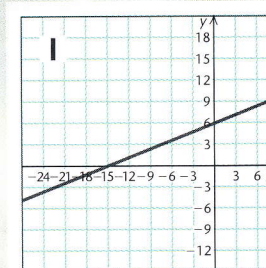
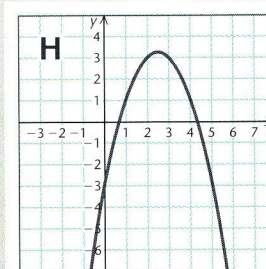
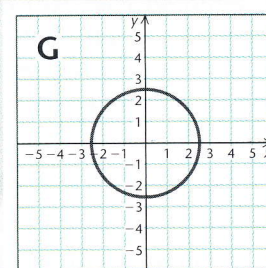
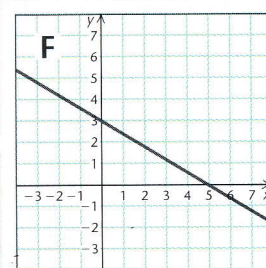
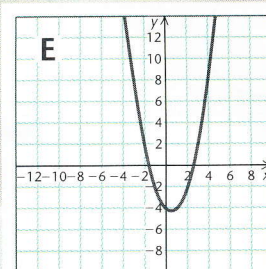
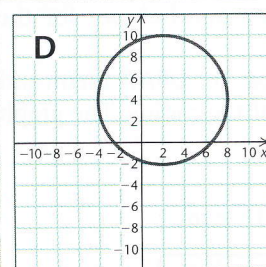
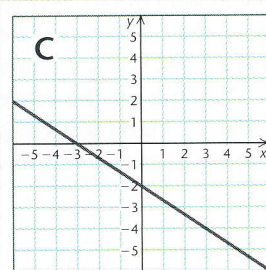
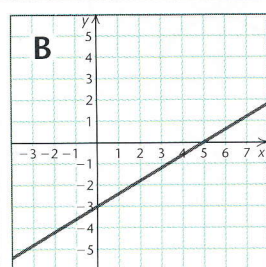
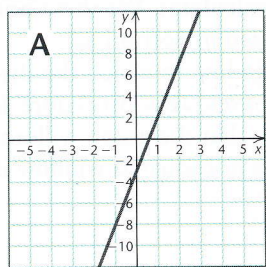
On many graphing calculators, the ratio of the height to the width of the viewing screen is $\frac{2}{3}$. When we choose a window in which $X_{\text{scl}} = Y_{\text{scl}}$ and the length of the y -axis is $\frac{2}{3}$ the length of the x -axis, the window will be squared. The windows with dimensions $[-6, 6, -4, 4]$, $[-9, 9, -6, 6]$, and $[-12, 12, -8, 8]$ are examples of squared windows. A graph of the circle $x^2 + y^2 = 36$ in a squared window is shown in Fig. 2. Many graphing calculators have an option on the ZOOM menu that squares the window automatically.

EXAMPLE 13 Graph the circle $(x - 2)^2 + (y + 1)^2 = 16$.

Solution The circle $(x - 2)^2 + (y + 1)^2 = 16$ has center $(2, -1)$ and radius 4, so the viewing window $[-9, 9, -6, 6]$ is a good choice for the graph.

To graph a circle, we select the CIRCLE feature from the DRAW menu and enter the coordinates of the center and the length of the radius. The graph of the circle $(x - 2)^2 + (y + 1)^2 = 16$ is shown here.





Visualizing the Graph

Match the equation with its graph.

- $y = -x^2 + 5x - 3$
- $3x - 5y = 15$
- $(x - 2)^2 + (y - 4)^2 = 36$
- $y - 5x = -3$
- $x^2 + y^2 = \frac{25}{4}$
- $15y - 6x = 90$
- $y = -\frac{2}{3}x - 2$
- $x^2 + y^2 + 6x - 2y - 6 = 0$
- $3x + 5y = 15$
- $y = x^2 - x - 4$

Answers on page A-4

1.1

Exercise Set

1, 5, 7-9, 13, 15-17, 23, 26, 30, 33, 34, 59, 60, 62
77, 78, 101-103, 106, 113, 114,

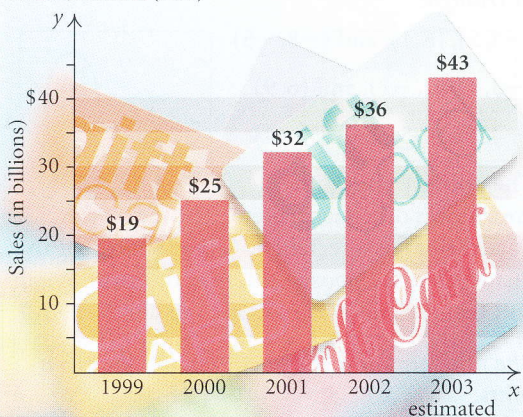
Graph and label the given points by hand.

1. $(4, 0)$, $(-3, -5)$, $(-1, 4)$, $(0, 2)$, $(2, -2)$
2. $(1, 4)$, $(-4, -2)$, $(-5, 0)$, $(2, -4)$, $(4, 0)$
3. $(-5, 1)$, $(5, 1)$, $(2, 3)$, $(2, -1)$, $(0, 1)$
4. $(4, 0)$, $(4, -3)$, $(-5, 2)$, $(-5, 0)$, $(-1, -5)$

Express the data pictured in the graph as ordered pairs, letting the first coordinate represent the year and the second coordinate the amount.

5.

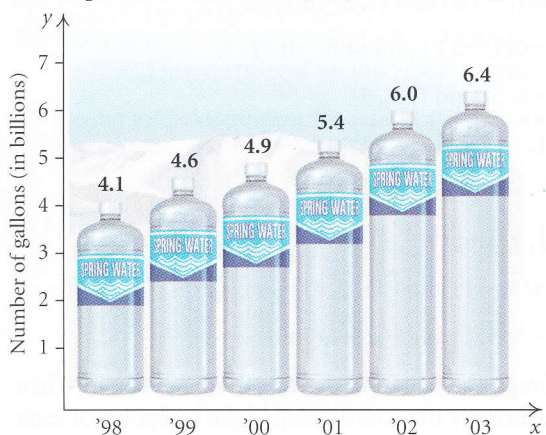
Gift Card Sales (U.S.)



Source: Bain and Co.

6.

Consumption of Bottled Water (U.S.)



Source: Beverage Marketing Corp.

Use substitution to determine whether the given ordered pairs are solutions of the given equation.

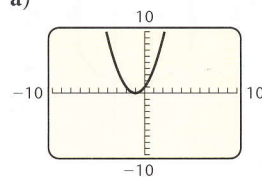
7. $(1, -1)$, $(0, 3)$; $y = 2x - 3$
8. $(2, 5)$, $(-2, -5)$; $y = 3x - 1$
9. $(\frac{2}{3}, \frac{3}{4})$, $(1, \frac{3}{2})$; $6x - 4y = 1$
10. $(1.5, 2.6)$, $(-3, 0)$; $x^2 + y^2 = 9$
11. $(-\frac{1}{2}, -\frac{4}{5})$, $(0, \frac{3}{5})$; $2a + 5b = 3$
12. $(0, \frac{3}{2})$, $(\frac{2}{3}, 1)$; $3m + 4n = 6$
13. $(-0.75, 2.75)$, $(2, -1)$; $x^2 - y^2 = 3$
14. $(2, -4)$, $(4, -5)$; $5x + 2y^2 = 70$

Use a graphing calculator to create a table of values with TBLSTART = -3 and $\Delta TBL = 1$. Then graph the equation by hand.

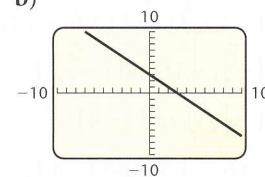
- | | |
|-----------------------------|----------------------------|
| 15. $y = 3x + 5$ | 16. $y = -2x - 1$ |
| 17. $x - y = 3$ | 18. $x + y = 4$ |
| 19. $2x + y = 4$ | 20. $3x - y = 6$ |
| 21. $y = -\frac{3}{4}x + 3$ | 22. $3y - 2x = 3$ |
| 23. $5x - 2y = 8$ | 24. $y = 2 - \frac{4}{3}x$ |
| 25. $x - 4y = 5$ | 26. $6x - y = 4$ |
| 27. $3x - 4y = 12$ | 28. $2x + 3y = -6$ |
| 29. $2x + 5y = -10$ | 30. $4x - 3y = 12$ |
| 31. $y = -x^2$ | 32. $y = x^2$ |
| 33. $y = x^2 - 3$ | 34. $y = 4 - x^2$ |
| 35. $y = -x^2 + 2x + 3$ | 36. $y = x^2 + 2x - 1$ |

In Exercises 37-40, use a graphing calculator to match the equation with one of the graphs (a)-(d), which follow.

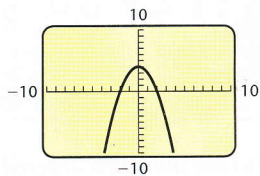
a)



b)



c)



37. $y = 3 - x^2$

39. $y = x^2 + 2x + 1$

Use a graphing calculator to graph the equation in the standard window.

41. $y = 2x + 1$

43. $4x + y = 7$

45. $y = \frac{1}{3}x + 2$

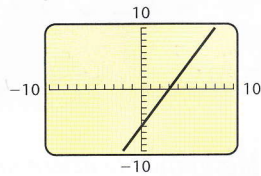
47. $2x + 3y = -5$

49. $y = x^2 + 6$

51. $y = 2 - x^2$

53. $y = x^2 + 4x - 2$

d)



38. $2x - y = 6$

40. $y = 4 - x^2$

42. $y = 3x - 4$

44. $5x + y = -8$

46. $y = \frac{3}{2}x - 4$

48. $3x + 4y = 1$

50. $y = x^2 - 8$

52. $y = 5 - x^2$

54. $y = x^2 - 5x + 3$

Graph the equation in the standard window and in the given window. Determine which window better shows the shape of the graph and the x - and y -intercepts.

55. $y = 3x^2 - 6$
[-4, 4, -4, 4]

56. $y = -2x + 24$
[-15, 15, -10, 30], with Xscl = 3 and Yscl = 5

57. $y = -\frac{1}{6}x^2 + \frac{1}{12}$
[-1, 1, -0.3, 0.3], with Xscl = 0.1 and Yscl = 0.1

58. $y = 8 - x^2$
[-3, 3, -3, 3]

Find the distance between the pair of points. Give an exact answer and, where appropriate, an approximation to three decimal places.

59. (4, 6) and (5, 9)

60. (-3, 7) and (2, 11)

61. (6, -1) and (9, 5)

62. (-4, -7) and (-1, 3)

63. (-4.2, 3) and (2.1, -6.4)

64. $(-\frac{3}{5}, -4)$ and $(-\frac{3}{5}, \frac{2}{3})$

65. $(-\frac{1}{2}, 4)$ and $(\frac{5}{2}, 4)$

66. (0.6, -1.5) and (-8.1, -1.5)

67. $(\sqrt{3}, -\sqrt{5})$ and $(-\sqrt{6}, 0)$

68. $(-\sqrt{2}, 1)$ and $(0, \sqrt{7})$

69. (0, 0) and (a, b)

70. (r, s) and (-r, -s)

71. The points (-3, -1) and (9, 4) are the endpoints of the diameter of a circle. Find the length of the radius of the circle.

72. The point (0, 1) is on a circle that has center (-3, 5). Find the length of the diameter of the circle.

The converse of the Pythagorean theorem is also a true statement: If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle. Use the distance formula and the Pythagorean theorem to determine whether the set of points could be vertices of a right triangle.

73. (-4, 5), (6, 1), and (-8, -5)

74. (-3, 1), (2, -1), and (6, 9)

75. (-4, 3), (0, 5), and (3, -4)

76. The points (-3, 4), (2, -1), (5, 2), and (0, 7) are vertices of a quadrilateral. Show that the quadrilateral is a rectangle. (Hint: Show that the quadrilateral's opposite sides are the same length and that the two diagonals are the same length.)

Find the midpoint of the segment having the given endpoints.

77. (4, -9) and (-12, -3)

78. (7, -2) and (9, 5)

79. (6.1, -3.8) and (3.8, -6.1)

80. (-0.5, -2.7) and (4.8, -0.3)

81. (-6, 5) and (-6, 8)

82. (1, -2) and (-1, 2)

83. $(-\frac{1}{6}, -\frac{3}{5})$ and $(-\frac{2}{3}, \frac{5}{4})$

84. $(\frac{2}{9}, \frac{1}{3})$ and $(-\frac{2}{5}, \frac{4}{5})$

85. $(\sqrt{3}, -1)$ and $(3\sqrt{3}, 4)$

86. $(-\sqrt{5}, 2)$ and $(\sqrt{5}, \sqrt{7})$

87. Graph the rectangle described in Exercise 76. Then determine the coordinates of the midpoint of each

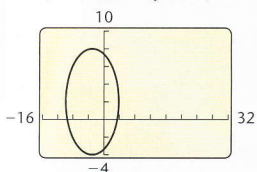
of the four sides. Are the midpoints vertices of a rectangle?

88. Graph the square with vertices $(-5, -1)$, $(7, -6)$, $(12, 6)$, and $(0, 11)$. Then determine the midpoint of each of the four sides. Are the midpoints vertices of a square?
89. The points $(\sqrt{7}, -4)$ and $(\sqrt{2}, 3)$ are endpoints of the diameter of a circle. Determine the center of the circle.
90. The points $(-3, \sqrt{5})$ and $(1, \sqrt{2})$ are endpoints of the diagonal of a square. Determine the center of the square.

In Exercises 91 and 92, how would you change the window so that the circle is not distorted? Answers may vary.

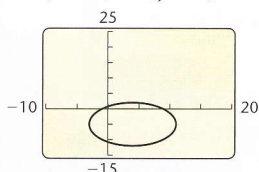
91.

$$(x + 3)^2 + (y - 2)^2 = 36$$



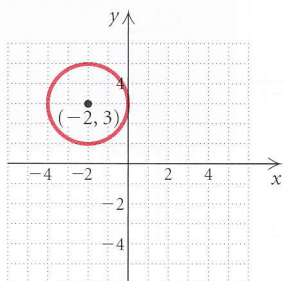
92.

$$(x - 4)^2 + (y + 5)^2 = 49$$



Find an equation for a circle satisfying the given conditions.

93. Center $(2, 3)$, radius of length $\frac{5}{3}$
94. Center $(4, 5)$, diameter of length 8.2
95. Center $(-1, 4)$, passes through $(3, 7)$
96. Center $(6, -5)$, passes through $(1, 7)$
97. The points $(7, 13)$ and $(-3, -11)$ are at the ends of a diameter.
98. The points $(-9, 4)$, $(-2, 5)$, $(-8, -3)$, and $(-1, -2)$ are vertices of an inscribed square.
99. Center $(-2, 3)$, tangent (touching at one point) to the y -axis



100. Center $(4, -5)$, tangent to the x -axis

Find the center and the radius of the circle. Then graph the circle by hand. Check your graph with a graphing calculator.

101. $x^2 + y^2 = 4$

102. $x^2 + y^2 = 81$

103. $x^2 + (y - 3)^2 = 16$

104. $(x + 2)^2 + y^2 = 100$

105. $(x - 1)^2 + (y - 5)^2 = 36$

106. $(x - 7)^2 + (y + 2)^2 = 25$

107. $(x + 4)^2 + (y + 5)^2 = 9$

108. $(x + 1)^2 + (y - 2)^2 = 64$

109. $x^2 + y^2 - 6x - 2y - 6 = 0$

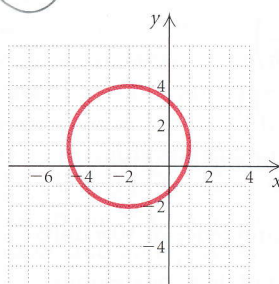
110. $x^2 + y^2 + 4x - 8y + 19 = 0$

111. $x^2 + y^2 + 2x + 2y = 7$

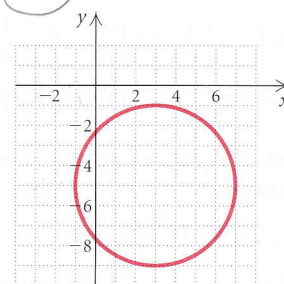
112. $x^2 + y^2 - 10x + 6y = -30$

Find the equation of the circle. Express the equation in standard form.

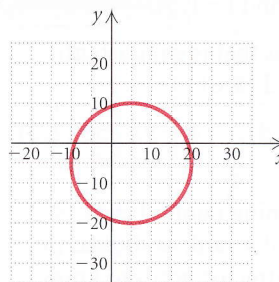
113.



114.



115.



116.

