Platonic Realms

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THE ESSENTIAL TRIGONOMETRY REFERENCE SHEET

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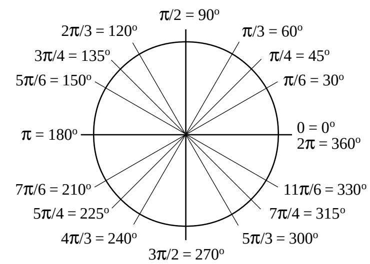
Essential Trigonometry Reference Sheet

TRIGONOMETRY evolved from the study of triangles, and has to do with both angle measurement and with certain natural functions defined on angles.

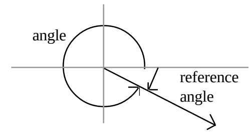
ANGLES Given two line segments with a common endpoint, the angle between them is defined to be the amount by which one segment must be rotated about the common endpoint to make it coincide with the other line segment. The angle is positive if the turning is counterclockwise, and negative if clockwise. If the common endpoint is the center of a circle, a right angle subtends ¼ of the circle, and a straight angle subtends ½ of the circle. In classical degree measure, a straight angle contains 180°. In modern radian measure, a straight angle contains π radians. Equivalently, an angle of 1 radian subtends an arc of the circle of arclength equal to the circle's radius.



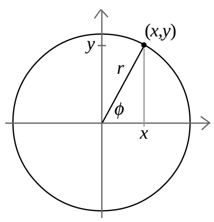
$$\frac{\text{radians}}{\text{degrees}} = \frac{\pi}{180}$$

RADIANS/DEGREES CONVERSION Use the formula at right, plugging in the known quantity and solving for the unknown quantity.

REFERENCE ANGLES It is often convenient to consider the reference angle of an angle; this is defined as the smallest angle to the terminating side from the horizontal (and may therefore be negative).

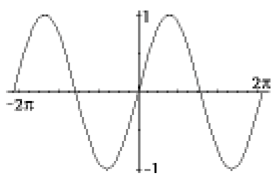


TRIG FUNCTIONS Referring to the figure, the six trigonometric functions are defined as indicated.

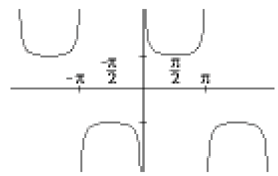


$$\sin\phi = \frac{y}{r} = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\cos\phi = \frac{x}{r} = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\sec\phi = \frac{r}{x} = \frac{\text{hypotenuse}}{\text{adjacent}}$$
$$\csc\phi = \frac{r}{y} = \frac{\text{hypotenuse}}{\text{opposite}}$$
$$\tan\phi = \frac{y}{x} = \frac{\text{opposite}}{\text{adjacent}}$$
$$\cot\phi = \frac{x}{y} = \frac{\text{adjacent}}{\text{opposite}}$$

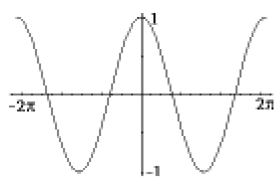
Sine



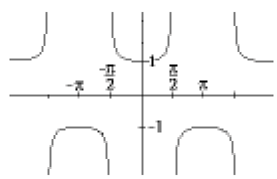
Cosecant



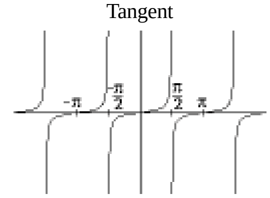
Cosine



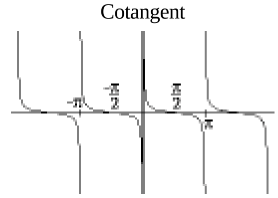
Secant



Tangent



Cotangent

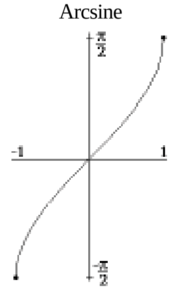


TRIG FUNCTION INVERSES Denoted either as arcsin or $\sin^{-1}$, etc., these satisfy

$$\sin\phi = x \;\leftrightarrow\; \phi = \sin^{-1}x$$
$$\cos\phi = x \;\leftrightarrow\; \phi = \cos^{-1}x$$
$$\tan\phi = x \;\leftrightarrow\; \phi = \tan^{-1}x$$

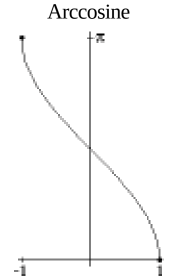
on their respective common domains, and similarly for arccsc, arcsec, and arccot.

Arcsine



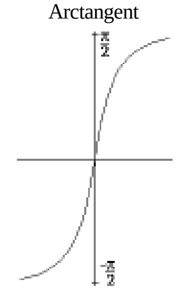
domain: [-1,1]
range: [-π/2,π/2]

Arccosine



domain: [-1,1]
range: [0,π]

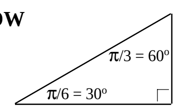
Arctangent

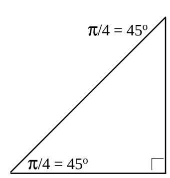


domain: (-∞,∞)
range: (-π/2,π/2)

SOME ANGLES GOOD TO KNOW

$$\sin(0) = \cos(\pi/2) = 0$$
$$\sin(\pi/2) = \cos(0) = 1$$
$$\sin(3\pi/2) = \cos(\pi) = -1$$
$$\sin(\pi/6) = \cos(\pi/3) = \tfrac{1}{2}$$
$$\sin(\pi/3) = \cos(\pi/6) = \tfrac{\sqrt{3}}{2}$$
$$\sin(\pi/4) = \cos(\pi/4) = \tfrac{\sqrt{2}}{2}$$
$$\tan(\pi/4) = \cot(\pi/4) = 1$$



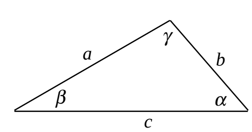


LAW OF SINES

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

LAW OF COSINES

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$



IDENTITIES & EQUIVALENCES

REDUCTION FORMULAS

$$\sin(\pi/2-\phi) = \cos\phi \qquad \sin(\pi/2+\phi) = \cos\phi$$
$$\cos(\pi/2-\phi) = \sin\phi \qquad \cos(\pi/2+\phi) = -\sin\phi$$
$$\tan(\pi/2-\phi) = \cot\phi \qquad \tan(\pi/2+\phi) = -\cot\phi$$
$$\sin(\pi-\phi) = \sin\phi \qquad \sin(\pi+\phi) = -\sin\phi$$
$$\cos(\pi-\phi) = -\cos\phi \qquad \cos(\pi+\phi) = -\cos\phi$$
$$\tan(\pi-\phi) = -\tan\phi \qquad \tan(\pi+\phi) = \tan\phi$$

FUNDAMENTAL EQUIVALENCES

$$\csc\phi = 1/\sin\phi$$
$$\sec\phi = 1/\cos\phi$$
$$\tan\phi = \sin\phi/\cos\phi$$
$$\cot\phi = \cos\phi/\sin\phi$$
$$\cot\phi = 1/\tan\phi$$

PYTHAGOREAN IDENTITIES

$$\sin^2\phi + \cos^2\phi = 1$$
$$1 + \tan^2\phi = \sec^2\phi$$
$$1 + \cot^2\phi = \csc^2\phi$$

DOUBLE ANGLE FORMULAS

$$\sin 2\phi = 2\sin\phi\cos\phi$$
$$\cos 2\phi = \cos^2\phi - \sin^2\phi = 1 - 2\sin^2\phi = 2\cos^2 - 1\phi$$
$$\tan 2\phi = \frac{2\tan\phi}{1+\tan^2\phi}$$

HALF ANGLE FORMULAS

$$\sin^2\phi = \frac{1-\cos 2\phi}{2}$$
$$\cos^2\phi = \frac{1+\cos 2\phi}{2}$$
$$\tan^2\phi = \frac{1-\cos 2\phi}{1+\cos 2\phi} = \frac{\sin^2 2\phi}{(1+\cos 2\phi)^2} = \frac{(1-\cos 2\phi)^2}{\sin^2 2\phi}$$

ANGLE SUM & DIFFERENCE

$$\sin(\phi+\psi) = \sin\phi\cos\psi + \cos\phi\sin\psi$$
$$\cos(\phi+\psi) = \cos\phi\cos\psi - \sin\phi\sin\psi$$
$$\tan(\phi+\psi) = \frac{\tan\phi+\tan\psi}{1-\tan\phi\tan\psi}$$
$$\cos(\phi-\psi) = \cos\phi\cos\psi + \sin\phi\sin\psi$$
$$\tan(\phi-\psi) = \frac{\tan\phi-\tan\psi}{1+\tan\phi\tan\psi}$$

FUNCTION PRODUCT, SUM, & DIFFERENCE

$$\sin\phi\cos\psi = \frac{1}{2}[\sin(\phi+\psi) + \sin(\phi-\psi)]$$
$$\cos\phi\sin\psi = \frac{1}{2}[\sin(\phi+\psi) - \sin(\phi-\psi)]$$
$$\cos\phi\cos\psi = \frac{1}{2}[\cos(\phi+\psi) + \cos(\phi-\psi)]$$
$$\sin\phi\sin\psi = -\frac{1}{2}[\cos(\phi+\psi) - \cos(\phi-\psi)]$$
$$\sin\phi + \sin\psi = 2\sin\frac{1}{2}(\phi+\psi)\cos\frac{1}{2}(\phi-\psi)$$
$$\sin\phi - \sin\psi = 2\cos\frac{1}{2}(\phi+\psi)\sin\frac{1}{2}(\phi-\psi)$$
$$\cos\phi + \cos\psi = 2\cos\frac{1}{2}(\phi+\psi)\cos\frac{1}{2}(\phi-\psi)$$
$$\cos\phi - \cos\psi = -2\sin\frac{1}{2}(\phi+\psi)\sin\frac{1}{2}(\phi-\psi)$$