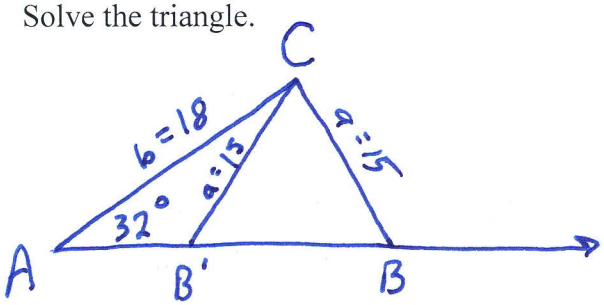
Worked Example – Law of Cosines: the ambiguous case

For triangle ABC with sides a, b, c: A = 32°; b = 18 cm; a = 15 cm.
Solve the triangle.



This is the ambiguous (S.S.A.) case. This means there might be 0, 1, or 2 solutions!

A diagram is always useful to estimate how many solutions you think you will get.

$a^2 = b^2 + c^2 - 2bc\cos A$

$15^2 = 18^2 + c^2 - 2(18)(c)\cos 32°$

$0 = c^2 - 30.52973c + 99$

Store (2)(18)(cos 32°) in your calculator STO▶

$$c = \frac{30.52\ldots \pm \sqrt{(30.52\ldots)^2 - 4 \cdot 1 \cdot 99}}{2}$$

Since the Law of Cosines is quadratic, it will provide 0, 1, or 2 solutions

Use the quadratic formula.

$c = 26.841399$ OR $c = 3.68833$

two solutions

$$\frac{15}{\sin 32°} = \frac{18}{\sin B} \rightarrow \sin B = \frac{18\sin 32}{15}$$

$B = 39.486999 \rightarrow C = 180° - A - B$

$C \approx 108.5°$

B could be obtuse (call it B')

$B' = 180° - 39.486\ldots \approx 140.513°$

$\rightarrow C' = 180° - A - B' \approx 7.5°$

There are 2 possible Δs

NOTE: Law of Sines only gives one angle -- and it is acute.
To find the other possible value (the obtuse angle) for ∠B use the isosceles triangle and the exterior angle.

Answers:

$B = 39.5°$	$B = 140.5°$
$C = 108.5°$	$C = 7.5°$
$c = 26.8$ cm	$c = 3.7$cm

Objective: Solving triangles using the Law of Cosines: the ambiguous case.