

Mental Math, Estimation, and Fact to Know Strategies

MM = Mental Math Strategy

ES = Estimation Strategy

FK = Fact to Know Strategy

1. [MM] **Counting On or Counting Back for Addition or Subtraction.** [A,S]
Examples: $472 + 3 = \text{“}472, 473, 474, 475\text{”} = 475$. $15,800 - 2,000 = \text{“}15,800, 14,800, 13,800\text{”} = 13,800$.
- 2.1 [MM] **To Multiply by 10, 100, 1000, etc.** Tack on Zeros or Move the Decimal Place (to the right because you are making it larger). [M]
Examples: $378 \times 100 = 37800$. $5.6 \times 1000 = 5600$.
- 2.2 [MM] **Divide by 10** [D]
Examples: $378 \div 10 = 37.8$. $37.8 \div 10 = 3.78$. $47,300 \div 10 = 4,730$.
- 2.3 [MM] **To Divide by 10, 100, 1000, etc.** Remove Zeros or Move the Decimal Place (to the left because you are making it smaller). [D]
Example: $3,600,000 \div 100 = 36,000$. $5.6 \div 1000 = .0056$. $780 \div 10000 = .078$.
2. [MM] **To Multiply or Divide by 10, 100, 1000, etc.** Tack on or remove zeros or move the decimal place [M, D]
Example: $470,000 \div 100 = 4,700$. $8.6 \times 1000 = 8600$.
- 3.1 [MM] **Front End Addition Strategy.** Perform the operation from left to right. [A]
Examples: $378 + 120 = 498$. $369 \div 9 = 41$. $48,320 \div 8 = 6040$.
- 3.2 [MM] **Front End Subtraction Strategy.** Perform the operation from left to right. [S]
Examples: $378 - 120 = 258$. $300 - 40 = 260$.
- 3.3 [MM] **Front End Multiplication Strategy.** Perform the operation from left to right. [M]
Examples: $3,010 \times 2 = 6,020$.
- 3.4 [MM] **Front End Division Strategy.** Perform the operation from left to right. [D]
Examples: $9663 \div 3 = 3221$. $369 \div 9 = 41$.
3. [MM] **Front End Strategy.** Perform the operation from left to right. [A,S,M,D]
Examples: $378 + 120 = 498$. $369 \div 9 = 41$. $48,320 \div 8 = 6,040$.
4. [FK] **Double any number up to 20.**
Examples: $18 \times 2 = 36$. $2 \times 17 = 34$.
- 5.1 [MM] **Compatible Numbers for Addition.** [A]
Examples: $25 + 8 + 6 + 12 + 25 = (25 + 25) + (8 + 12) + 6 = 50 + 20 + 6 = 76$.
- 5.2 [MM] **Compatible Numbers for subtraction.** [S]
Examples: $32 + 16 - 12 - 6 = (32 - 12) + (16 - 6) = 20 + 10 = 30$.
- 5.3 [MM] **Compatible Numbers for multiplication.** [M]
Examples: $2 \times 8 \times 5 \times 7 = (2 \times 5) \times (8 \times 7) = 10 \times 56 = 560$.
5. [MM] **Compatible Numbers Strategy.** [A,S,M]
Examples: $25 + 8 + 6 + 12 + 25 = (25 + 25) + (8 + 12) + 6 = 50 + 20 + 6 = 76$. $2 \times 8 \times 5 \times 7 = (2 \times 5) \times (8 \times 7) = 10 \times 56 = 560$.
6. [MM] **Break Apart.** [A,S,M] (For division, use “Front End.”)
Examples: $7 \times 12 = 7 \times 10 + 7 \times 2 = 70 + 14 = 84$. $3 \times 42 = 3 \times 40 + 3 \times 2 = 120 + 6 = 126$. $215 - 83 = (210 - 80) + (5 - 3) = 130 + 2 = 132$.
7. [MM] **To Divide by 5 Double it and Divide by 10.** [D]
Example: $28 \div 5 = 5.6$.
8. [MM] **Compensation.** [A,M]
Examples: $18 \times 5 = 20 \times 5 - \text{“}compensation\text{”} = 20 \times 5 - 2 \times 5 = 100 - 10 = 90$. $65 + 38 = 65 + 40 - \text{“}compensation\text{”} = 105 - 2 = 103$.

9. [MM] **Equal Additions Technique for Subtracting.** [S]
Example: $74 - 28 = 76 - 30 = 46$.
10. [MM] **Solve Proportions with Doubling or Halving** [M,D]
Example: Solve $\frac{x}{7} = \frac{20}{14}$, $x = 10$.
- 11.1 [MM] **Drop the Zeros, Multiply, Then Put Them Back On.** [M]
Example: $300 \times 50 = 15000$.
- 11.2 [MM] **Drop the Zeros, Add or Subtract, Then Put Them Back On.** [A,S]
Example: $300 + 50 = 350$. $4,200 - 300 = 3,900$.
11. [MM] **Drop the Zeros, Add, Subtract, or Multiply, Then Put Them Back On.** [A,S, M]
Example: $300 \times 60 = 18000$. $5,200 - 300 = 4,900$.
12. [FK] **Halve the even numbers from 30 to 40.**
Examples: $38 \div 2 = 19$. Half of 34 is 17.
13. [ES] **Rounding** [A,S,M,D]
Example: $28,319 - 1,978 \approx 28,000 - 2,000 = 26,000$.
14. [ES] **Substitute Compatible Numbers** [A,S,M,D]
Example: $38 \times 391 \times 26 \approx 40 \times 25 \times 331 = 1000 \times 331 \approx 391,000$. (Note that this is *not rounding*. You may not substitute for all numbers. That is, some numbers may be used as is.)
15. [FK] **Double any number up to 50**
Examples: $28 \times 2 = 56$. $2 \times 47 = 94$.
16. [ES] **Front-End Estimation with Adjustment.** [A,S,M,D]
Examples: $3185 \times 203 \approx 3000 \times 200 \approx 600000 + \text{"adjustm't"} \approx 640000$. (When adjusting addition, consider the numbers you *did not use*, only adjust *up*, and always adjust the right-most place you used (which may be the *only* place you used!)). $859 + 221 + 112 + 523 \approx 800 + 200 + 100 + 500 + \text{"adjustm't"} = 1600 + \text{"numbers not used do justify an increase of 100"} = 1600 + 100 \approx 1700$.
17. [ES] **Clustering Technique for Addition.** [A]
Example: $712 + 699 + 694 \approx 700 \times 3 \approx 2100$.
18. [MM] **10% of a Number** [M]
Example: 10% of 12 = 1.2; 10% of 23,400 = 2,340.
19. [MM] **Adding with 5's in the Units Place.** [A]
Example: $35 + 35 = 70$. $45 + 25 = 70$.
20. [MM] **Cancel Zeros, Drop Some of the Zeros, Divide, Then "Put Them Back On."** [D]
Example: $30000 \div 50 = 30 \div 5$ with two zeros = 600.
21. [FK] **Part-Part-Whole on 100.** [A,S]
Example: $100 - 41 = 59$.
22. [FK] **Multiply, and Divide 11 & 12.** [M,D]
Example: $12 \times 7 = 84$. $6 \times 11 = 66$.
- 23.1 [MM] **Multiply by 4 by doubling twice**
Examples: $378 \div 10 = 37.8$. $37.8 \div 10 = 3.78$. $47,300 \div 10 = 4,730$.
- 23.2 [MM] **Divide by 4 by dividing by 2 twice**
Examples: $378 \div 10 = 37.8$. $37.8 \div 10 = 3.78$. $47,300 \div 10 = 4,730$.
23. [MM] **To Multiply or Divide by 4, Use 2 Twice.** [M,D]
Example: $4 \times 35 = 140$. $500 \div 4 = 125$. $84 \div 4 = 21$.
24. [MM] **Think Multiplication Method of Dividing.** [D]
Example: $350 \div 7 = 50$. $48 \div 12 = 4$.
25. [MM] **Use Doubling when 2 is a Factor.** [M]
Example: $6 \times 35 = 3 \times (2 \times 35) = 3 \times 70 = 210$. $8 \times 13 = 104$.
26. [FK] **Halve any even number ≤ 100 .**

Examples: $68 \div 2 = 34$. Half of 74 is 37.

27. [MM] **Multiply by a Unit Fraction by Dividing.** [M]
Examples: $\left(\frac{1}{5}\right) 35 = 35 \div 5 = 7$.
28. [FK] **Part-Part-Whole on 60 (an hour).** [A,S]
Example: $60 - 43 = 17$. At 7:48, how many minutes is it until 8:00?
- 29.1 [MM] **Multiplying Decimals by dropping the decimal and putting it back in later.** [M]
Examples: $.80 \times .3 = 24$. $.5 \times .3 = .15$.
- 29.2 [MM] **Dividing by Decimals by dropping the decimal and putting it back in later.** [D]
Example: $.36 \div .3 = 120$.
29. [MM] **Multiplying or Dividing by Decimals by dropping the decimal and putting it back in later.** [M, D]
Examples: $800 \times .3 = 240$. $12 \div .3 = 40$.
30. [FK] **Part-Part-Whole on 90 (complementary angle).** [A,S]
Example: $90 - 21 = 69$.
31. [MM] **Solve Proportions using Factor of Change** [M,D]
Example: Solve $\frac{2}{9} = \frac{a}{36}$, $a = 8$.
32. [FK] **Part-Part-Whole on 180 (supplementary angle).** [A,S]
Example: $180 - 21 = 159$.
33. [FK] **Know Basic Percent-Fraction-Decimal Equivalences-multiples of 25%.** (0%, 25%, 50%, 75%, 100%)
Example: $75\% = \frac{3}{4} = .75$.
34. [FK] **Know Basic Percent-Fraction-Decimal Equivalences-multiples of 10%.** (10%, 20%, 30%, ... , 90%)
Example: $60\% = .6 = 6/10 = 3/5$.
35. [MM] **Adding on to Find a Difference (the Change).** [S]
Example: Cost is \$7.55. \$10 paid. Change is \$2.45.
36. [MM] **To Multiply by 5 Multiply by 10 and divide by 2.** [M]
Example: $28 \times 5 = 140$.
37. [FK] **Skip count by 15.**
Example: Continue the pattern: 60, 75, 90,
38. [MM] **Find 25, 50, and 75% of a Number.** [M]
Example: 50% of 80 is 40. 25% of 80 is 20. 75% of 80 is 60.
39. [ES] **Find Estimated Percents by Substituting Compatible Numbers.** [M,D]
Example: $17/48 \approx 16/48 = \frac{1}{3} = 25\%$. $9/26 \approx 8/24 = 1/3 \approx 33\%$ (or $9/26 \approx 9/27 = 1/3 \approx 33\%$)
40. [FK] **Part-Part-Whole on 1 with a Fraction** [A,S]
Example: $1 - 3/8 = 5/8$. $1 - 23/40 = 17/40$.
41. [MM] **Think of Quarters.** [A,S,M,D]
Example: $175 + 25 = 200$. $250 - 75 = 175$. $25 \times 5 = 125$. $325 \div 25 = 13$.
42. [FK] **Cube all numbers up to 5** [M]
Examples: $3^3 = 27$; $5^3 = 125$.
43. [MM] **Solve Proportions using the Unit Rate Method** [M,D]
Example: Solve $\frac{15}{3} = \frac{a}{4}$, $15 \div 3 = 5$, $5 \times 4 = 20 = a$.
44. [MM] **Percent of a Number with One Non-zero Digit.** [M]
Example: 34% of 2,000 = 680.
45. [MM] **Make Compatible Factors.** [M]
Example: $28 \times 25 = 7 \times (4 \times 25) = 700$. $15 \times 36 = (15 \times 4) \times 9 = 60 \times 9 = 540$.
46. [MM] **To Multiply by a Multiple of 10% Multiply the Digits and Put the decimal Point in the Reasonable Place.** [M]

- Example:* 70% of 80 is 56. 80% of 800 is 640. 40% of 7 = 2.8.
47. [MM] **To Multiply by a Percent with one non-zero digit Multiply the Digits and Put the decimal Point in the Reasonable Place.** [M]
Example: 300% of 60 is 180. 70% of 80 is 56. 3% of 70 = 2.1.
48. [MM] **Use an Intermediate Value When Adding on to Find a Difference.** [S]
Example: Cost is \$0.45. \$1 paid. 5¢ to get to 50¢ plus 50¢ to get to a dollar. Change is 0.55.
 415 – 230: Use intermediate value of 300. 70 to get to 300, plus 115 (to get from 300 to 415),
 415 – 230 = 70 + 115 = 185.
49. [MM] **Fraction of a Number by Reducing** (including multiplying by a unit fraction). [M]
Example: $\frac{1}{3}(60) = 20$. $\frac{4}{7}(42) = 24$.
50. [MM] **Mixed Number Times a Number using Break Apart.** [M]
Example: $1\frac{1}{2}(60) = 90$. $2\frac{1}{3}(90) = 210$.
51. [MM] **Use a Helping Fact.** [A, S, M, D]
Example: $78 \div 3 = 75 \div 3$ “plus one more” = 26.
52. [MM] **To Multiply by a decimal with one non-zero digit Multiply the Digits and Put the decimal Point in the Reasonable Place.** [M]
Example: $.3 \times 60$ is 18. $.03 \times 60$ is 1.8. $.005 \times 70,000$ is 350.
53. [MM] **Use Order of Operations to Zoom and Focus .** [A, S, M, D]
Example: Find $f(4)$, for $f(x) = 5(x - 7)^2 - 1$. Find $g(0)$, for $g(x) = 7x^2 - 9x + 2$.
54. [MM] **Find 5, 10, and 15% of a Number.** [M]
Example: 10% of 80 is 8. 5% of 80 is 4. 15% of 80 is 12.
55. [MM] **Find 20% of a Number.** [M]
Example: 20% of 80 is 16. 20% of 62 is 12.4.
56. [MM] **Multiply a Number Times a Percent Greater than 100% - like a Mixed Number.** [M,D]
Example: 110% of 70 is 77. 125% of 24 is 30.
57. [MM] **“Teens” Times Table** [M]
Example: $16 \times 17 = 16+7$ and put on a 0 and add $6*7 = 230 + 42 = 272$
58. [FK] **Know Basic Percent-Fraction-Decimal Equivalences-multiples of $33\frac{1}{3}\%$.** ($33\frac{1}{3}\%$ and $66\frac{2}{3}\%$)
Example: What is $33\frac{1}{3}\%$ as a fraction? What is $66\frac{2}{3}\%$ as a decimal?
59. [MM] **Find $33\frac{1}{3}\%$ and $66\frac{2}{3}\%$ of a Number.** [M]
Example: $33\frac{1}{3}\%$ of 24 is 8. $66\frac{2}{3}\%$ of 60 is 40.
60. [FK] **Part-Part-Whole on 1 with a Decimal** [A,S]
Example: $1 - .21 = .79$.
61. [MM] **Adding on to Find a Difference in Time.** [S]
Example: 4:15 to 6:00 is 1:45.
62. [MM] **Equal Additions Technique for Subtracting Decimals.** [S]
Example: $75.2 - 3.7 = 75.5 - 4.0 = 71.5$.
63. [MM] **Equal Additions Technique for Subtracting Fractions.** [S]
Example: $7\frac{1}{8} - \frac{7}{8} = 7\frac{2}{8} - \frac{8}{8} = 7\frac{2}{8} - 1 = 6\frac{2}{8} = 6\frac{1}{4}$.
64. [FK] **Powers of 2 up to a K (1024).** [M]
Example: 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024.
65. [FK] **Use Difference of Squares.** [M]
Example: $19 \times 21 = 20^2 - 1^2 = 399$. $57 \times 63 = 60^2 - 3^2 = 3600 - 9 = 3591$.
66. [MM] **Find Percents by Using Equivalent Fractions with a Denominator of 100** [M,D]
Example: $\frac{17}{20} = \frac{85}{100} = 85\%$
67. [ES] **Estimate Percents by Using Equivalent Fractions with a Denominator of 100** [M,D]
Example: $\frac{16}{35} \approx \frac{15}{33} \approx \frac{45}{100} = 45\%$

68. [MM] **Squaring Numbers Ending in 5** [M]
Example: $85^2 = 8 \times 9$ “and” $25 = 7,225$. $35^2 = 3 \times 4$ “and” $25 = 1,225$.
69. [FK] **Part-Part-Whole on 360 (for trigonometry)** [A,S]
Example: An angle of 310° is how many degrees from the x-axis?
70. [FK] **Know Basic Percent-Fraction-Decimal Equivalences-multiples of 12.5%.** ($1/8, 3/8, 5/8, 7/8$)
Examples: What is $1/8$ as a percent? (12.5%) What is $3/8$ as a decimal? (.375)
71. [MM] **Find 12.5% of a Number.** [M]
Examples: 12.5% of 80 is 10. 12.5% of 320 is 40.
72. [FK] **Part-Part-Whole on π (for trigonometry)** [A,S]
Examples: $\pi - \pi/4 = 3\pi/4$. $\pi - 5\pi/6 = \pi/6$.
73. [FK] **Relationships involving $\sin^2 x$ and $\cos^2 x$**
Examples: $1 - \sin^2 x = \cos^2 x$. $\cos^2 x - 1 = -\sin^2 x$.
74. [FK] **Relationships involving the other two Pythagorean Identities**
Examples: $\csc^2 x = 1 + \cot^2 x$. $\tan^2 x + 1 = \sec^2 x$.
75. [FK] **Decimal approximations for multiples of π** [M,D]
Examples: Estimate: $\pi \approx 3.14$. $\pi/2 \approx 1.57$.
76. [FK] **Decimal approximations for multiples of square root of 2 and square root of 3** [M,D]
Examples: Estimate: $\text{root } 2 \approx 1.414$. $\cos(30^\circ) \approx .866$.
77. [MM] **To multiply or divide by 8, use 2 three times.** [M,D]
Examples: $8 \times 35 = 280$. $200 \div 8 = 25$.
78. [MM] **To multiply by .25, 2.5, 25, 250 use one-quarter.** [M]
Examples: $.25 \times 80 = 20$. $250 \times 280 = 70,000$.
79. [MM] **To multiply by .75, 7.5, 75, 750 use three-quarters.** [M]
Examples: $.75 \times 800 = 600$. $7.5 \times 280 = 2,100$.
80. [FK] **Take half of a fraction or mixed number.** [M]
Examples: $\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$; $\frac{1}{2} \cdot \frac{4}{5} = \frac{2}{5}$; $\frac{1}{2} \cdot (14\frac{3}{4}) = 7\frac{3}{8}$.
81. [FK] **Halve (almost) any even number using front end.** [D]
Examples: $268 \div 2 = 134$. Half of 834 is 417. $261,804 \div 2 = 130,902$.