

Fraction Review ~ Yellow Pages

Notation for Multiplication

Multiplication is indicated by use of parentheses, a cross, a dot, an asterisk, or putting symbols next to each other.

Examples: $5 \times 2 = (5)(2) = 5 \cdot 2 = 5 * 2 = 10$,

$5 \times x = (5)(x) = 5 \cdot x = 5 * x = 5x$.

Notation for Division

Division is indicated by use of the fraction bar, slash or division sign. Examples: $\frac{120}{30} = 120/30 = 120 \div 30 = 4$.

Note on dividing by 1: It may be useful to express a whole number (e.g., 5), as a fraction. That is, the number over 1 (e.g., $\frac{5}{1}$). This is true because $\frac{5}{1} = 5 \div 1 = 5$.

Fractions

A **fraction** is a rational number expressed in the form $\frac{a}{b}$ (which is the preferred notation) or a/b (in-line notation), where a is called the **numerator** and b is called the **denominator**. The **fraction bar** means **divided by**.

Examples: $\frac{a}{b} = a/b = a \div b$, $\frac{15}{5} = 15 \div 5 = 3$.

Caution: Special care should be used when the in-line notation is used. Parentheses should be put around the numerator and denominator.

$\frac{x}{yz} = x/(yz)$, but $\frac{x}{yz} \neq x/yz$ (because, in x/yz , the order of operations would perform the division first, followed by the multiplication).

Notes on Zero in Fractions

If **0 (zero) is in the numerator** of a fraction (and a non-zero number is in the denominator), then the fraction is 0.

Example: $\frac{0}{5} = 0$.

It is not legal to have **0 in the denominator**, because we may not divide by zero. Example: $\frac{4}{0}$ is not a legal fraction.

The Many Forms of One as a Fraction

When the numerator of a fraction is equivalent to the denominator of a fraction, the value of the fraction is 1.

Examples: $\frac{5}{5} = 1$, $1 = \frac{7}{7}$, $\frac{16\text{ounces}}{1\text{pound}} = 1$, $\frac{1\text{km}}{1000\text{m}} = 1$. This is very useful for rewriting fractions and converting units.

Putting a Fraction in Lower (lowest) Terms

To rewrite a fraction in lower terms find a common factor of the numerator and denominator and divide the numerator and denominator by the common factor. (This process is often referred to a "simplifying.") This process may be repeated. If there is no common factor of the numerator and denominator, then the fraction is in *lowest terms*. Examples:

$\frac{6}{15} = \frac{6 \div 3}{15 \div 3} = \frac{2}{5}$, $\frac{36}{48} = \frac{36 \div 2}{48 \div 2} = \frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$.

Rewriting an Improper Fraction into a Mixed Number

An "**improper fraction**" is a fraction whose numerator is greater than or equal to the denominator. Improper fractions are greater than or equal to 1. To rewrite an improper

$$\begin{array}{r} 3 \frac{4}{5} \\ 5 \overline{) 19} \\ \underline{-15} \\ 4 \end{array}$$

fraction into a mixed number (a **mixed number** has a whole number part and a fractional part), perform the long division and put the remainder over the divisor. Example: $\frac{19}{5} = 3 \frac{4}{5}$.

Multiplying Fractions

To multiply two fractions:

1. Multiply the numerators.
2. Multiply the denominators.
3. Put the result in lowest terms.

Alternately, you may *simplify first*, by dividing a common factor into a numerator and denominator, then follow steps 1-3. (If you simplified completely initially, it will eliminate the need to do step 3.) Examples: $\frac{4}{9} \times \frac{7}{10} = \frac{28}{90} = \frac{14}{45}$,

$\frac{4}{9} \times \frac{7}{10} = \frac{2}{9} \times \frac{7}{5} = \frac{14}{45}$, $7 \times \frac{5}{11} = \frac{7}{1} \times \frac{5}{11} = \frac{35}{11} = 3 \frac{2}{11}$.

Rewriting a Mixed Number into an Improper Fraction

It is often useful to express a mixed number as an improper fraction. To do this,

1. Express the whole number as a fraction with the same denominator as the fractional part
2. Add the fractions.

Example: $4 \frac{3}{5} = \frac{4}{1} + \frac{3}{5} = (\frac{4}{1})(\frac{5}{5}) + \frac{3}{5} = \frac{20}{5} + \frac{3}{5} = \frac{23}{5}$.

Putting a Fraction in Higher Terms

To rewrite a fraction in higher terms, you multiply by 1, in the form of $\frac{a}{a}$. In effect, you will multiply the numerator and denominator by the same number. Examples:

$\frac{5}{7} = (\frac{5}{7})(\frac{3}{3}) = \frac{15}{21}$, $\frac{2x}{7} = (\frac{2x}{7})(\frac{4}{4}) = \frac{8x}{28}$.

Adding Fractions

To add fractions, the denominators must be equal. (If the fractions already have common denominators, then skip step #1.) To add fractions:

1. Rewrite each fraction so they each have a **common denominator**. Do this by multiplying each fraction by a form of 1 ($\frac{3}{3}$, for example). To determine what the least common denominator (LCD) should be, determine the least common multiple of the original denominators.
2. Add the numerators of the fractions.
3. The denominator will be the common denominator. of the built-up fractions.
4. Put the answer in lowest terms.

Examples: $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$,

$\frac{1}{4} + \frac{5}{6} = \frac{1}{4}(\frac{3}{3}) + \frac{5}{6}(\frac{2}{2}) = \frac{3}{12} + \frac{10}{12} = \frac{13}{12} = 1 \frac{1}{12}$,

$\frac{3}{5} + \frac{1}{2} + \frac{1}{6} = \frac{3}{5}(\frac{6}{6}) + \frac{1}{2}(\frac{15}{15}) + \frac{1}{6}(\frac{5}{5}) = \frac{18}{30} + \frac{15}{30} + \frac{5}{30} = \frac{38}{30} = 1 \frac{4}{15}$

Subtracting Fractions

To subtract fractions:

Use the steps for adding fractions, but subtract the numerators in step #2.

Example: $\frac{13}{24} - \frac{3}{8} = \frac{13}{24} - \frac{9}{24} = \frac{4}{24} = \frac{1}{6}$.

Dividing Fractions

To divide two fractions:

1. Take the reciprocal of the divisor (the second fraction).
2. Change the operation to multiplication.
3. Follow the steps above for multiplying fractions.

$$\text{Examples: } \frac{3}{7} \div \frac{4}{5} = \frac{3}{7} \times \frac{5}{4} = \frac{15}{28},$$

$$\frac{4}{9} \div \frac{14}{3} = \frac{4}{9} \times \frac{3}{14} = \frac{2}{3} \times \frac{1}{7} = \frac{2}{21}.$$

Multiplying and Dividing Mixed Numbers

To multiply or divide mixed numbers:

1. Rewrite each mixed number as an improper fraction.
2. Perform the operation as stated above.

$$\text{Examples: } 1\frac{1}{3} \times 4\frac{2}{5} = \frac{4}{3} \times \frac{22}{5} = \frac{88}{15} = 5\frac{13}{15},$$

$$6 \times 5\frac{1}{2} = \frac{6}{1} \times \frac{11}{2} = \frac{66}{2} = 33,$$

$$1\frac{1}{3} \div 4\frac{2}{5} = \frac{4}{3} \div \frac{22}{5} = \frac{4}{3} \times \frac{5}{22} = \frac{20}{66} = \frac{10}{33},$$

$$\frac{4}{7} \div 3 = \frac{4}{7} \div \frac{3}{1} = \frac{4}{7} \times \frac{1}{3} = \frac{4}{21}, \quad 3 \div \frac{4}{7} = \frac{3}{1} \div \frac{4}{7} = \frac{3}{1} \times \frac{7}{4} = \frac{21}{4}.$$

Adding Mixed Numbers

To add mixed numbers:

1. Add the whole numbers.
2. Add the fractional parts (as stated above).
3. If the fractional part is an improper fraction, regroup (express it as a mixed number and add to the whole number).

$$\text{Examples: } 5\frac{1}{4} + 2\frac{5}{12} = 7\frac{2}{3},$$

$$8\frac{4}{5} + 7\frac{3}{5} = 16\frac{2}{5}.$$

$$\begin{array}{r} 5\frac{1}{4} \rightarrow 5\frac{3}{12} \\ + 2\frac{5}{12} \rightarrow + 2\frac{5}{12} \\ \hline 7\frac{8}{12} = 7\frac{2}{3} \end{array}$$

$$\begin{array}{r} 8\frac{4}{5} \\ + 7\frac{3}{5} \\ \hline 15\frac{7}{5} = 16\frac{2}{5} \end{array}$$

Subtracting Mixed Numbers

To subtract mixed numbers:

1. You'll want to subtract fractional parts, so first get a common denominator.

Furthermore, if the fraction you are subtracting is larger than the first fraction, you need to regroup one from the whole number, and then subtract.

2. Subtract the whole numbers.

3. Put the answer in lowest terms.

$$\text{Examples: } 11\frac{2}{3} - 7\frac{1}{6} = 4\frac{1}{2}, \quad 16\frac{2}{9} - 12\frac{2}{3} = 3\frac{5}{9}.$$

$$\begin{array}{r} 11\frac{2}{3} \rightarrow 11\frac{4}{6} \\ - 7\frac{1}{6} \rightarrow - 7\frac{1}{6} \\ \hline 4\frac{3}{6} = 4\frac{1}{2} \end{array}$$

$$\begin{array}{r} 16\frac{2}{9} \rightarrow 16\frac{2}{9} \rightarrow 15\frac{11}{9} \\ - 12\frac{2}{3} \rightarrow - 12\frac{6}{9} \rightarrow - 12\frac{6}{9} \\ \hline 3\frac{5}{9} \end{array}$$

Complex Fractions

A complex fraction is a fraction which has fractions in the numerator and/or denominator. We'll call these fractions inside the "little fractions." To simplify a complex fraction,

- Determine the LCD of the little fractions,
- Multiply numerator and denominator by this LCD
- Use the distributive property and simplify.

$$\text{Example: } \frac{\frac{5}{3} + x}{4 - \frac{1}{2}} = \frac{(\frac{5}{3} + x) 6}{(4 - \frac{1}{2}) 6} = \frac{10 - 6x}{24 - 3} = \frac{10 - 6x}{21}$$

Decimals

Decimals are a base-10 notational system which can be used to represent fractions. The whole number part is to the left of the decimal and the fractional part is to the right.

Place Values to the Right of the Decimal Point									
		Tenths	Hundredths	Thousandths	Ten-Thousandths	Hundred-Thousandths	Millionths	Ten-Millionths	...
0	.	1	2	3	4	5	6	7	...

Examples: 0.09 is read, "nine hundredths."

35.7 is read, "thirty-five and seven tenths."

6.519 is read, "six and five hundred nineteen thousandths."

$$0.09 = \frac{9}{100}, \quad 35.7 = 35\frac{7}{10}, \quad 6.519 = 6\frac{519}{1000}.$$

Percents – Decimals – Fractions

Percent means per 100, which means divided by 100. There are three equivalent forms of a number: fraction, decimal, and percent.

Conversions of Percents, Decimals, and Fractions

Percent to Decimal: divide by 100, which moves the decimal point to the left two places. $23\% = .23$; $156.7\% = 1.567$; $2\% = .02$; $45.7\% = .457$

Decimal to Percent: move the decimal point to the right two places and put on the percent sign. $.23 = 23\%$; $.02 = 2\%$; $1.343 = 134.3\%$; $.00089 = .089\%$; $.712 = 71.2\%$

Fraction to Decimal: Do the long division. All fractions will have a terminating or repeating decimal form.

$$\text{Examples: } \frac{3}{4} = 3 \div 4 = .75,$$

$$\frac{9}{8} = 9 \div 8 = 1.125.$$

$$\frac{8}{11} = 8 \div 11 = 0.727272... = 0.\overline{72}.$$

Decimal to Fraction: Read the decimal (examples above). Put in lowest terms. (Put the mixed number into an improper fraction as needed.)

Percent to Fraction and Fraction to

Percent [Method 1]: Convert to a decimal first, then convert to the other form. Examples: $\frac{3}{4} = .75 = 75\%$,

$$\frac{9}{8} = 1.125 = 112.5\%, \quad 2.8\% = .028 = \frac{28}{1000} = \frac{7}{250}.$$

Fraction to Percent [Method 2]: Use a proportion with the fraction on one side and $\frac{p}{100}$ is on the other side, where p is the percent. Solve the proportion. Example:

$$\frac{3}{4} = \frac{p}{100} \rightarrow 4p = 300 \rightarrow p = \frac{300}{4} = 75, \text{ so } \frac{3}{4} = 75\%.$$

Percent to Fraction [Method 2]: Use the fraction $\frac{p}{100}$, where p is the percent. If p has a decimal, multiply numerator and denominator to make both a whole number. Finally, put in

$$\text{lowest terms. Examples: } 75\% = \frac{75}{100} = \frac{3}{4},$$

$$2.8\% = \frac{2.8}{100} = \frac{2.8(10)}{100(10)} = \frac{28}{1000} = \frac{7}{250}.$$

