HOW MUCH DIVERSIFICATION IS ENOUGH? –

WELFARE IMPLICATIONS FOR INVESTORS

UNDER UNCERTAINTY

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Abstract

Welfare losses are inevitable for investors should they decide to invest sub-optimally. To measure those welfare losses I compare *n*-asset optimal portfolios with *26*-asset optimal portfolios by using the concept of the proportionate opportunity cost along with various CRRA utility functions. The original historical asset returns are used with a VAR in generating joint returns distributions for the portfolio formation period. In each case 1,000 alternative sets of assets including one with a risk-free nominal return are randomly made available for investment. I show that the well-diversified number of assets is 24. A sub-optimal number of assets (less than 24) generates opportunity costs for investors. The opportunity cost decreases as the level of risk aversion increases and as the number of assets in portfolios increases. The opportunity cost also increases when investors use a restricted VAR to derive joint returns distribution function.

HOW MUCH DIVERSIFICATION IS ENOUGH? – WELFARE IMPLICATIONS FOR INVESTORS UNDER UNCERTAINTY.

1. *Introduction*

The key question in the literature regarding well-diversified portfolios and the number of assets in portfolios is: at what point it is no longer very helpful to make more assets available for the portfolio? How can an investor tell whether his portfolio is well-diversified? Is there a substantial difference between diversification with the number of assets in an investor's portfolio constrained to be less than optimal investment strategy suggests (i.e. sub-optimally diversified portfolio) and optimal diversification?

Cheng and Liang (2000) address the last of those questions and found that there is evidence to support the idea that optimally diversified portfolios are more efficient than sub-optimally diversified portfolios in the context of mean-variance framework. To test the efficiency difference they set up and test the hypothesis that the Sharpe ratio for an efficient portfolio equals the Sharpe ratio of a sub-optimally diversified portfolio. The question that is left unanswered in the paper and the question that I am interested in is: how inefficient is a sub-optimally diversified portfolio relative to an optimally diversified one?

In order to answer the question I will compare expected utility from the optimal portfolio constrained to include *n* assets with that from the optimal unconstrained portfolio permitted to have 26 assets, as an approximation of an infinite number of assets that gives the highest diversification gain, by using the concept of opportunity cost. Then I will show how this comparison varies with *n*.

At a certain n I will find that further diversification is no longer very helpful: the opportunity cost of investing in these n assets rather than in 26 assets does not exceed 1% of initial wealth. Under the condition stated above the n will be defined as a well-diversified number of assets.

The proportionate opportunity cost is the best way to measure investors' welfare losses from any kind of constraint on their holdings. Under the assumption of the constant relative risk aversion utility function

$$(1) \qquad U(\widetilde{w}) = \begin{cases} \dfrac{1}{\gamma}\widetilde{w}^{\gamma}, \ \gamma < 1, \ \gamma \neq 0, \ \widetilde{w} > 0 \\ -\infty, \ \widetilde{w} \leq 0 \end{cases}$$

the proportionate opportunity cost (willingness to accept payment as compensation for being constrained to only n assets) can be calculated as $\theta - 1.0$ where θ is defined by

$$(2) \qquad EU(\theta w_0 \widetilde{R}_n^{optimal}) = EU(w_0 \widetilde{R}_{26}^{optimal})$$

where w_0 is initial wealth, $\widetilde{R}_{26}^{optimal}$ and $\widetilde{R}_n^{optimal}$ are the stochastic returns per dollar invested for the portfolios with 26 and n assets respectively. Solving (2) with the utility function (1) gives

$$(3) \qquad \theta = \left[\frac{E(\widetilde{R}_{26}^{\gamma})^{optimal}}{E(\widetilde{R}_n^{\gamma})^{optimal}}\right]^{\frac{1}{\gamma}}.$$

The proportionate opportunity cost, $\theta - 1.0$, is timeless. But its numerical value depends on a number of months until horizon, i.e. with the investment horizon of T months the proportionate willingness to accept payment to accept the constraint is θ^T.

One motivation for finding the cost of sub-optimal diversification comes from Kelly (1995) which showed that "…[t]hree quarters of the households in the top quintile

(of the survey sample) of stock ownership had fewer than ten different stocks". It raises the possibility that U.S. household behavior may not be well-diversified in terms of reducing idiosyncratic risk. The question arises: what is their cost of not diversifying well?

Brennan and Torous (1999) have addressed the issue of the cost of sub-optimal diversification, and Fama (1972) and Sankaran and Patil (1999) have addressed the issue of how many securities is enough for a well-diversified portfolio.

Brennan and Torous worked with a constant relative risk aversion utility function and with the certainty equivalent concept. They used the certainty equivalent as a way to measure the investor's loss when he diversifies sub-optimally. The authors randomly picked starting years and the securities for portfolios from CRSP. To form a portfolio they used the equally-weighted-portfolio rule (equal number of dollars invested in every asset). They formed portfolios with different numbers of assets in them. Then they calculated expected utility for those portfolios using a constant relative risk aversion utility function. The whole process of choosing a starting year, drawing securities, forming portfolios and calculating expected utility was repeated 10,000 times. Then, for every portfolio the certainty equivalent was calculated. And this certainty equivalent showed how much an investor would lose if he diversified sub-optimally (should he not have enough assets in his portfolio, not enough to call his portfolio well-diversified). The authors found that there are still significant welfare gains for an investor to be received even when the number of securities in the investor's portfolio is as high as 20.

Brennan and Torous's equal-weighting rule of constructing portfolios is not appealing. The $\frac{1}{n}$ rule is characterized in the literature as a "naïve" portfolio strategy

(Kroll et al. (1984)). The more appealing approach, which I will follow here, will be to use optimal portfolios that can be found through an optimization procedure. The comparison I am going to use will not be "equally-weighted-portfolio for n assets versus equally-weighted-portfolio for 26 assets" but rather "optimal portfolio with n assets versus optimal portfolio with 26 assets". Furthermore, my approach differs from theirs in terms of generating the ex ante returns distribution for the investment period. I use a vector autoregressive process (VAR) to project the means of returns and to capture 120 historically occurring shocks to all asset returns; then I assume that the true distribution of shocks for the investment period is given by adding those 120 sets of returns shocks with equal probabilities to the conditional means.

Working with the mean-variance theory Fama (1972) looked at the relationship between the standard deviation of a portfolio return and the number of assets in the portfolio. What he found is "…[m]ost of the effects of diversification … occur when the first few securities are added to the portfolio. Once the portfolio has 20 securities, further diversification has little effect". To construct his portfolios Fama used randomly selected stocks and in his framework the cost of not investing in the optimal number of securities is measured in terms of the standard deviation of the portfolio return. He found that as the number of stocks in portfolios increases, the standard deviation of portfolio return decreases.

In his approach Fama (1972), like Brennan and Torous (1999), used the $\frac{1}{n}$ rule to construct all portfolios in order to do all his calculations and comparisons. And as was mentioned before, the existing literature characterizes the rule as a "naïve" portfolio strategy (Kroll et al. (1984)). And the rule is not appealing. The more appealing approach

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is to use optimal portfolios (not mean-variance efficient as Fama did, but rather globally optimal) that can be found through an optimization procedure.

Sankaran and Patil (1999), working with the same mean-variance theory, found that: "…[d]iversification beyond 8-10 securities may not be worthwhile". But this specific number of securities, as the authors pointed out, comes from optimally selecting a security to be constrained to zero quantity rather than randomly choosing a security to be excluded from the portfolio. In any event, their conclusion is: as the number of stocks in one's portfolio increases that will significantly reduce the risk of underperforming inflation and the stock market, and of losing portfolio value (Vassal (2001)).

The procedure used in the present paper, for calculating the proportionate opportunity cost for an investor of being constrained by the number of assets in his portfolio, includes random asset selection for investors' portfolios, estimation of a vector autoregressive process, derivation of the joint probability distribution function of asset returns, and computing optimal portfolios.

In the first part of the paper I show that with a nominally risk-free asset, the well-diversified number of assets in one's portfolio depends on degree of risk aversion and on the way a VAR process was used in deriving the asset returns distribution functions for the purpose of evaluating the opportunity cost. I found that the largest well-diversified number of assets in one's portfolio is 24. As relative risk aversion increases the well-diversified number of assets in one's portfolio decreases. It is a very counterintuitive conclusion but a clear reason emerges. The results also show that for investors with high levels of risk aversion the well-diversified number of assets is less then three due to the fact that they place more than 90% of their initial wealth into Treasury bills. In the case

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with the unrestricted VAR the well-diversified number of assets is larger than that for the restricted VAR.

The second section of the paper describes the procedure of random asset selection for investors' portfolios, of inferring the joint probability distribution function of asset returns via a vector autoregression, of computing the constrained optimal and unconstrained optimal portfolios, and of the calculation of the proportionate opportunity cost. The third section discusses the results. The fourth section concludes.

2. *The Procedure*

2.1. *Asset Selection*

The procedure of calculating the proportionate opportunity cost for various *n* for each of various levels of risk aversion will be performed 1,000 times, in each case using randomly picked nominally risky assets and Treasury bills as the nominally risk-free asset.

I compute opportunity costs for each degree of risk aversion with and without a short-selling constrained in a sequence of rounds 1, 2, 3, … corresponding to *n=3, 4, 5, … .* The first round is to pick at random 25 nominally risky assets. The unconstrained optimal portfolio, with 26 assets, will include all 25 nominally risky assets and Treasury bills as the nominally risk-free asset. The constrained optimal portfolio, with *n=3* assets, will include two nominally risky assets that I will pick at random from the 25 originally randomly picked nominally risky assets, and Treasury bills. To construct the optimal constrained portfolio and optimal unconstrained portfolio expected values of real returns for time *T+1* are needed for all nominally risky assets and for nominally risk-free

Treasury bills. In real terms, though, there is no risk-free asset. Returns on Treasury bills are risk-free only in nominal terms. But in time-series data inflation will be uncertain in any period and, thus, so will the real rate of return on Treasury bills. Therefore, the 26 assets and the three assets will all be risky assets in real terms.

2.2. *Vector Autoregressions of Returns*

To get expected values of real returns for the case of 26 assets and three assets at time $T+1$, the portfolio formation period, I estimate a vector autoregressive process (VAR). The next steps are to derive the joint probability distribution for the two groups of assets' real returns, and, finally, to construct optimal constrained and optimal unconstrained portfolios.

To derive the joint probability distribution of empirical deviations from the VAR-estimated conditional means for those 25 randomly picked asset returns and inflation the following procedure is done.

The nominal return on asset i at time t minus the nominal return on Treasury bills at time t gives us the excess return on asset i ($x_{i,t}$) at time t for $i=1,...,25$ and for $t=1, ...,T$. A VAR for excess returns of those 25 assets and realized inflation is

$$(4) \quad \begin{bmatrix} x_{1,t} \\ . \\ . \\ x_{25,t} \\ \pi_t \end{bmatrix} = \begin{bmatrix} c_1 \\ . \\ . \\ c_{25} \\ c_{26} \end{bmatrix} + \begin{bmatrix} v_{1,1}(L) & . & . & v_{1,26}(L) \\ . & . & . & . \\ v_{25,1}(L) & . & . & v_{25,26}(L) \\ v_{26,1}(L) & . & . & v_{26,26}(L) \end{bmatrix} \begin{bmatrix} x_{1,t} \\ . \\ . \\ x_{25,t} \\ \pi_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ . \\ . \\ \varepsilon_{25,t} \\ \varepsilon_{\pi,t} \end{bmatrix},$$

delivering $\{\hat{c}_i\}$, $\{\hat{\varepsilon}_{i,t}\}$ and $\{\hat{v}_{i,k}(L)\}$, where

$$(5) \quad \hat{v}_{i,k}(L) = \hat{\delta}_{i,k}^1 L^1 + \hat{\delta}_{i,k}^2 L^2 + ...$$

The vector of conditional expected values of excess returns for time $T+1$ and expected inflation for time $T+1$ is

(6)
$$
\begin{bmatrix} E_T x_{1,T+1} \\ . \\ E_T x_{25,T+1} \\ E_T \pi_{T+1} \end{bmatrix} = \begin{bmatrix} \hat{c}_1 \\ . \\ \hat{c}_{25} \\ \hat{c}_{26} \end{bmatrix} + \begin{bmatrix} \hat{v}_{1,1}(L) & . & . & \hat{v}_{1,26}(L) \\ . & . & . & . \\ \hat{v}_{25,1}(L) & . & . & \hat{v}_{25,26}(L) \\ \hat{v}_{26,1}(L) & . & . & \hat{v}_{26,26}(L) \end{bmatrix} \begin{bmatrix} x_{1,T+1} \\ . \\ x_{25,T+1} \\ \pi_{T+1} \end{bmatrix}.
$$

The expected real return on asset i in period $T+1$, the portfolio formation period, is

(7)
$$
\begin{bmatrix} E_T r_{1,T+1} \\ . \\ E_T r_{25,T+1} \end{bmatrix} = \begin{bmatrix} E_T x_{1,T+1} \\ . \\ E_T x_{25,T+1} \end{bmatrix} + \begin{bmatrix} r^n_{TB,T+1} \\ . \\ r^n_{TB,T+1} \end{bmatrix} - \begin{bmatrix} E_T \pi_{T+1} \\ . \\ E_T \pi_{T+1} \end{bmatrix}
$$

where $r^n_{TB,T+1}$ is the ex ante observed nominal return on Treasury bills for time $T+1$. The expected real return on Treasury bills for time $T+1$ is

(8)
$$
E_T r_{TB,T+1} = r^n_{TB,T+1} - E_T \pi_{T+1}.
$$

Finally, the conditional probability distribution for real returns for time $T+1$ is determined by

(9)
$$
\begin{bmatrix} \tilde{r}_{1,T+1} \\ . \\ \tilde{r}_{25,T+1} \\ \tilde{r}_{TB,T+1} \end{bmatrix} = \begin{bmatrix} E_T x_{1,T+1} \\ . \\ E_T x_{25,T+1} \\ 0 \end{bmatrix} + \begin{bmatrix} r^n_{TB,T+1} \\ . \\ r^n_{TB,T+1} \\ r^n_{TB,T+1} \end{bmatrix} - \begin{bmatrix} E_T \pi_{T+1} \\ . \\ E_T \pi_{T+1} \\ E_T \pi_{T+1} \end{bmatrix} + \begin{bmatrix} \tilde{\varepsilon}_{1,T+1} \\ . \\ \tilde{\varepsilon}_{25,T+1} \\ 0 \end{bmatrix} - \begin{bmatrix} \tilde{\varepsilon}_{\pi,T+1} \\ . \\ \tilde{\varepsilon}_{\pi,T+1} \\ \tilde{\varepsilon}_{\pi,T+1} \end{bmatrix}
$$

where $\begin{bmatrix} \tilde{\varepsilon}_{1,T+1} \\ . \\ \tilde{\varepsilon}_{25,T+1} \\ \tilde{\varepsilon}_{\pi,T+1} \end{bmatrix}$ takes on the historically observed values $\begin{bmatrix} \tilde{\varepsilon}_{1,t} \\ . \\ \tilde{\varepsilon}_{25,t} \\ \tilde{\varepsilon}_{\pi,t} \end{bmatrix}$ from regression (4),

$t=1,2,...,T$, with equal probabilities $(1/T)$.

This way of deriving asset returns probability distribution functions, using historically occurring innovations to asset returns captured through this VAR procedure,

is superior to the VAR method mentioned in the literature, e.g. Campbell and Viceira (2002). The literature on derivation of asset returns probability distribution functions assumes that the distribution of asset returns is static, not evolving over time. But the reality is such that the asset returns distribution is dynamic, depending on both recent realizations and the fixed historical distribution of shocks to the dynamic asset returns process. So the right way of deriving asset returns probability distribution functions is to include the dynamics of the past history of asset returns.

Similarly, to get the probability distribution of returns for use in *three*-asset portfolios, the above procedure including the VAR is redone using (4)-(9) with 26 changed to three.

Therefore, in the first part of the paper two absolutely different VAR are used to derive the joint asset returns distributions for 26 and for three assets.

2.3. *Constrained Portfolios*

Using the information about those three assets' derived probability distribution for their real returns (computed using the equations analogous to (4)-(9)), I compute the constrained optimal portfolio with *n=3* assets: the solution of

$$(10) \qquad \underset{\{\alpha_1,\alpha_2\}}{Max}\, EU(\widetilde{w}) = Max\; E\left\{\frac{1}{\gamma}\left[w_0\left(\alpha_1\widetilde{r}_1 + \alpha_2\widetilde{r}_2 + \left(1-\alpha_1-\alpha_2\right)\widetilde{r}_{TB}\right)\right]^{\gamma}\right\}$$

where α_1, α_2, $1-\alpha_1-\alpha_2$ are the three individual assets' portfolio shares in the constrained optimal portfolio. To get the portfolio I search over α_1, α_2 space to optimize expected utility, using nonlinear optimization by a quasi-Newton method based on convergence to

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first-order conditions of problem (10). The expectation is taken over the joint probability distribution derived as described above analogously to (4)-(9).

2.4. *Unconstrained portfolios*

The next step, then, is to get the unconstrained optimal portfolio with 26 assets: the solution of

$$(11) \qquad \underset{\{\alpha_1,\dots,\alpha_{25}\}}{Max} EU(\tilde{w}) = Max\ E\left\{\frac{1}{\gamma}\left[w_0\left(\alpha_1\tilde{r}_1 + \dots + \alpha_{25}\tilde{r}_{25} + \left(1 - \alpha_1 - \dots - \alpha_{25}\right)\tilde{r}_{TB}\right)\right]^\gamma\right\}$$

where $\alpha_1, \dots, \alpha_{25}$ are the first 25 individual assets' portfolio shares in the unconstrained optimal portfolio. To get the portfolio I search over $\alpha_1, \dots, \alpha_{25}$ space to optimize expected utility, again using nonlinear optimization by a quasi-Newton method based on convergence to first-order conditions of problem (11). This time, the expectation is taken over the joint probability distribution derived as described above in (4)-(9).

2.5. *Calculating Opportunity Cost*

Having computed the constrained and unconstrained optimal portfolios I can now calculate the opportunity cost, θ-1.0. For the formula for θ, (3), I need to find $E(\tilde{R}_{26}^\gamma)^{\text{optimal}}$, where $\tilde{R}_{26}$ is the gross return for the optimal unconstrained portfolio with 26 assets, and $E(\tilde{R}_n^\gamma)^{\text{optimal}}$, where $\tilde{R}_n$ is the gross return for the constrained optimal portfolio with three assets.

$E(\tilde{R}_{26}^\gamma)^{\text{optimal}}$ (referring more completely to $E_T(\tilde{R}_{26,T+1}^\gamma)^{\text{optimal}}$) is equal to

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$$(12) \quad E_T(\tilde{R}^{\gamma}_{26,T+1})^{optimal} = \frac{1}{T}\sum_{t=1}^{T}\left\{\begin{bmatrix}\alpha_1^{**} & ... & \alpha_{25}^{**} & 1-\alpha_1^{**}-...-\alpha_{25}^{**}\end{bmatrix}\begin{bmatrix}E_T r_{1,T+1}+\varepsilon_{1,t}-\varepsilon_{\pi,t} \\ . \\ E_T r_{25,T+1}+\varepsilon_{25,t}-\varepsilon_{\pi,t} \\ E_T r_{TB,T+1}+0-\varepsilon_{\pi,t}\end{bmatrix}\right\}^{\gamma}$$

where the vector of α_i^{**} is the vector of optimal portfolio shares for the portfolio with 26 assets; the vectors of $E_T r_{i,T+1} + \varepsilon_{i,t} - \varepsilon_{\pi,t}$ and $E_T r_{TB,T+1} - \varepsilon_{\pi,t}$ are the vectors of particular possible values of real returns (conditional on the data set for times $t=1$ through T) at time $T+1$ (the portfolio formation period) and calculated as shown in (4)-(9).

And E($\tilde{R}_n^{\gamma}$)^{optimal} is equal to

$$(13) \quad E_T(\tilde{R}^{\gamma}_{3,T+1})^{optimal} = \frac{1}{T}\sum_{t=1}^{T}\left\{\begin{bmatrix}\alpha_1^{*} & \alpha_2^{*} & 1-\alpha_1^{*}-\alpha_2^{*}\end{bmatrix}\begin{bmatrix}E_T r_{1,T+1}+\varepsilon_{1,t}-\varepsilon_{\pi,t} \\ E_T r_{2,T+1}+\varepsilon_{2,t}-\varepsilon_{\pi,t} \\ E_T r_{TB,T+1}+0-\varepsilon_{\pi,t}\end{bmatrix}\right\}^{\gamma}$$

where α_i^{*} is the vector of optimal portfolio shares for the portfolio with n assets. In (13) the expectations are taken over the distribution implied by the n-asset (in this case 3-asset) VAR. Thus the θ calculations will reflect only the cost of restricting the number of assets. Subsequently the θ calculations will be redone, taking the expectations in (13) over the distribution implied by the 26-asset VAR. These θ's will also reflect the cost of having chosen the n-asset portfolios using a restricted size of the VAR.

Then, having calculated (12) and (13), the (3) is used to get a numerical value for θ. And the proportionate opportunity cost for an investor of investing in three assets rather than in 26 assets is simply θ - 1.0.

The second round starts by retaining the 25 originally picked nominally risky assets and Treasury bills. This time, again, the 26-asset portfolio will be the

unconstrained optimal portfolio, and $E(\widetilde{R}_{26}^{\gamma})^{\text{optimal}}$ is equal to $E(\widetilde{R}_{26,T+1}^{\gamma})^{\text{optimal}}$ as already

computed. The constrained optimal portfolio this time will include four assets including

Treasury bills, and to get that I pick three nominally risky assets at random from the

original 25. Then, to get expected values of real returns for the four assets including

Treasury bills for time $T+1$ I estimate a new *four*-variable vector autoregressive process.

After that the joint probability distribution for the four assets' real returns is derived, and

the optimal constrained four-asset portfolio is constructed where $E(\widetilde{R}_{n}^{\gamma})^{\text{optimal}}$ is equal to

$E(\widetilde{R}_{4,T+1}^{\gamma})^{\text{optimal}}$. Finally, using (13) with n updated to four I calculate θ and, hence, the

proportionate opportunity cost of investing in four assets rather than in 26 assets.

    For each round, the procedure is repeated 1,000 times. This produced 1,000

values of θ for each n. I do this (round after round) until the mean value of the

proportionate opportunity cost of investing in n assets rather than in 26 is no larger than

1%. The entire procedure is done for each of 11 alternative values of the risk aversion

parameter γ.

    To estimate the proportionate opportunity costs that reflect not only the cost of

restricting the number of assets in investors' portfolios but also the cost of using a

restricted VAR, the entire above procedure including the derivation of VAR is redone

where only a *26*-asset VAR was used. Therefore, the expectations in (12) and (13) were

taken over the distributions of asset returns implied by the same *26*-asset VAR for each

set of available assets.

    The second part of the paper discusses the results of using only one VAR for

derivation of the asset returns distributions for 26 assets as well as for n assets.

3. *Results*

The results from this research project are as follows.

3.1. *Results Derived for Opportunity Cost of Restricting the Number of Assets in Portfolios.*

This part of the section discusses the results derived from calculations of the proportionate opportunity costs that reflect only the cost of restricting the number of assets in investors' portfolios. In the course of calculating these costs, *n*-asset and *26*-asset VARs were used. Therefore, the expectations in (12) and (13) were taken over the distributions of asset returns implied by the two different VARs.

3.1.1. *Opportunity Costs*

Table 1 presents the results from calculation of 1,000 values of the proportionate opportunity cost for each of 11 different values of relative risk aversion for portfolios with three to 25 assets based on returns distributions derived from historically occurring asset returns over the 10-year period January 1994 through December 2003.

For relative risk aversion of 0.7 the number of assets beyond which the mean value (over 1,000 replications) of the opportunity cost is no higher than 1% of initial wealth is 24. With 24 assets in the investor's portfolio for the risk aversion level of 0.7 it is no longer helpful to make more assets available for the portfolio: Table 1 reports that the mean value of the proportionate opportunity cost of investing in 25 assets rather than in 26 is 0.4% (0.004). As risk aversion increases, though, the well-diversified number of assets in investors' portfolio decreases. It reaches 20 assets for risk aversion of three; nine

Table 1

The proportionate opportunity cost of restricting the number of assets in portfolios, (θ-1), for various values of relative risk aversion (mean values over 1,000 replications)

Number of assets in portfolios	0.7	1	2	3	9	10	11	12	29	30	31
3	0.165	0.118	0.062	0.041	0.013	0.012	0.012	0.011	**0.006**	**0.005**	**0.004**
4	0.159	0.092	0.061	0.039	0.012	0.012	0.011	0.011	0.005	0.003	0.002
5	0.157	0.091	0.058	0.038	0.012	0.011	0.011	0.011	0.005	0.003	0.001
6	0.147	0.084	0.056	0.036	0.011	0.011	0.011	**0.010**	0.003	0.001	0.001
7	0.141	0.081	0.053	0.034	0.011	0.011	**0.010**	0.007	0.003	0.001	0.000
8	0.135	0.080	0.051	0.032	0.011	**0.010**	0.008	0.007	0.001	0.000	0.000
9	0.126	0.078	0.049	0.030	**0.010**	0.009	0.007	0.006	0.001	0.000	0.000
10	0.119	0.077	0.045	0.029	0.009	0.008	0.007	0.005	0.000	0.000	0.000
11	0.113	0.075	0.042	0.027	0.008	0.007	0.005	0.005	0.000	0.000	0.000
12	0.105	0.073	0.040	0.026	0.007	0.007	0.005	0.005	0.000	0.000	0.000
13	0.098	0.069	0.037	0.024	0.007	0.006	0.005	0.003	0.000	0.000	0.000
14	0.090	0.063	0.034	0.023	0.006	0.005	0.003	0.001	0.000	0.000	0.000
15	0.083	0.057	0.031	0.020	0.006	0.005	0.003	0.001	0.000	0.000	0.000
16	0.076	0.054	0.029	0.019	0.006	0.005	0.003	0.001	0.000	0.000	0.000
17	0.071	0.051	0.027	0.017	0.005	0.005	0.001	0.000	0.000	0.000	0.000
18	0.063	0.044	0.024	0.015	0.005	0.004	0.001	0.000	0.000	0.000	0.000
19	0.054	0.040	0.021	0.011	0.005	0.003	0.000	0.000	0.000	0.000	0.000
20	0.047	0.035	0.018	**0.010**	0.004	0.003	0.000	0.000	0.000	0.000	0.000
21	0.037	0.028	0.012	0.007	0.003	0.000	0.000	0.000	0.000	0.000	0.000
22	0.028	0.018	**0.010**	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.019	**0.010**	0.006	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	**0.010**	0.006	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.004	0.030	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

assets for risk aversion of nine; and three or less assets for investors with risk aversion of 29 and higher.

As Table 1 shows, the highest mean of the proportionate opportunity cost, 16.5%, corresponds to the lowest level of risk aversion of 0.7 and will be incurred by an investor should he decide to invest in three assets rather than in 26. The lowest mean of the proportionate opportunity cost, 0.0%, corresponds to the highest level of risk aversion of 31 and will be incurred by an investor should he decide to invest in seven or more assets rather than in 26.

The mean values of the proportionate opportunity cost of investing in four assets rather than 26 range from 15.9% for the level of risk aversion of 0.7 to 0.2% for the level of risk aversion of 31. The mean values of the proportionate opportunity cost of investing in five assets rather than 26 range from 15.7% for the level of risk aversion of 0.7 to 0.1% for the level of risk aversion of 31. The mean values of the proportionate opportunity cost of investing in ten assets rather than in 26 range from 11.9% for the level of risk aversion of 0.7 to 0.0% for the level of risk aversion of 31. It is clear that the mean values of the proportionate opportunity cost of investing in n assets rather than in 26 decrease as the number of assets available for investment increases, and as the level of relative risk aversion increases.

3.1.2. *Optimal portfolio shares*

Table 2 presents typical optimal portfolio shares for unconstrained (*26*-asset) and constrained portfolio strategies for three different levels of relative risk aversion: low (of 0.7), medium (of 11) and high (of 31). For all three levels of risk aversion there is a different well-diversified number of assets; therefore, constrained optimal portfolios for different levels of risk aversion have different numbers of assets in them. In each case illustrative optimal portfolio shares are calculated for a different set of available assets whose number is just large enough to give an opportunity cost of no more than 1.0%, the cut-off value after which the diversification benefits are assumed to be non-significant.

For Table 2 for risk aversion of 0.7 more than 100% of initial wealth, w_0, is held in the nominally risky assets (asset #1 through asset #25) as a group, and Treasury bills are held in negative quantities.

Table 2

Illustrative optimal portfolio shares for unconstrained and optimally constrained to include *n* assets portfolio strategies for different values of relative risk aversion[1]

Asset #	Relative Risk Aversion,$(1-\gamma)$, equal to 0.7		Relative Risk Aversion, $(1-\gamma)$, equal to 11		Relative Risk Aversion, $(1-\gamma)$, equal to 31	
	Unconstrained	Constrained	Unconstrained	Constrained	Unconstrained	Constrained
1	3.055	2.764	0.049	0.000	0.001	0.000
2	-0.081	-0.129	0.024	0.006	0.049	0.023
3	0.337	0.000	0.076	0.000	-0.081	0.000
4	2.621	2.145	-1.000	0.000	0.016	0.000
5	-0.427	-0.502	0.066	0.000	0.166	0.088
6	0.149	0.008	0.195	0.000	0.001	0.000
7	0.638	0.664	0.031	0.182	0.035	0.000
8	-0.019	-0.385	-0.026	0.000	0.049	0.000
9	-1.991	-2.197	-0.014	0.000	0.008	0.000
10	-0.305	-0.309	0.049	0.000	0.018	0.000
11	0.647	0.544	-0.042	0.000	0.066	0.000
12	1.004	1.191	0.097	0.089	-0.029	0.000
13	1.456	1.515	0.007	0.002	0.016	0.000
14	0.119	0.095	-0.082	0.000	-0.007	0.000
15	-0.572	-0.396	0.201	0.191	-0.049	0.000
16	0.137	0.289	0.053	0.000	-0.082	0.000
17	-1.837	-1.827	0.091	0.000	0.021	0.000
18	0.001	0.172	-0.025	0.000	0.005	0.000
19	0.266	0.294	0.059	0.000	0.018	0.000
20	-0.997	0.000	0.022	0.000	-0.003	0.000
21	0.294	0.256	-0.149	-0.059	-0.009	0.000
22	0.479	0.574	0.086	0.000	0.031	0.000
23	0.551	0.661	-0.046	0.000	-0.016	0.000
24	0.430	0.367	0.042	0.000	0.021	0.000
25	1.497	1.257	0.070	0.000	0.024	0.000
26[2]	-6.451	-6.051	0.265	0.589	0.736	0.889
$E(X^{*\prime}\widetilde{R})$ [3]	1.166	1.156	1.018	1.007	1.007	1.000
Certainty Equivalent	1.088	1.077	1.009	0.999	1.001	0.993

[1] *Numbers are not comparable across levels of risk aversion, because for each level of risk aversion a different set of available assets was used.*
[2] *The 26th asset is risk-free in nominal terms.*
[3] *Monthly gross expected returns on portfolios.*

As risk aversion increases, as investors become more conservative and less risk-tolerant, the proportion of initial wealth held in Treasury bills becomes positive and increases, and correspondingly the proportion of initial wealth held in the group of nominally risky assets decreases.

The tables show that unconstrained and constrained optimal portfolio shares are not similar for different levels of risk aversion. As a matter of fact, optimal unconstrained and constrained portfolios for the low level of relative risk aversion of 0.7 have more extreme quantities (negative as well as positive) of assets than optimal unconstrained and constrained portfolios for the medium level of relative risk aversion of 11 and for the high level of relative risk aversion of 31. Extremely negative quantities of assets for high risk-tolerance investors mean that the investors follow an aggressive short sale strategy.

Table 2 also shows monthly expected returns on unconstrained and constrained optimal portfolios, $E\ (X^{*\prime}\widetilde{R})$, for the three levels of relative risk aversion. The expected returns for constrained and unconstrained optimal portfolios for risk aversion of 0.7 are very dramatic. Expected returns are of medium size for risk aversion of 11 and of small size for risk aversion of 31. Such magnitudes of expected portfolio returns for high risk-tolerance investors confirm the previously made conclusion about very aggressive short sale strategies. These magnitudes suggest very leveraged portfolios (unconstrained as well as constrained). For investors with risk aversion of 11 and 31 there is, definitely, some short selling going on as well. The less aggressive short selling for medium or high risk aversion leads to lower mean return portfolios.

Table 2 also reports the *certainty equivalents* calculated for the same three levels of relative risk aversion (0.7, 11 and 31). The *certainty equivalent*, (*CE*), is defined by

$$(15) \qquad \frac{1}{\gamma} CE^{\gamma} = \frac{1}{\gamma} w_0^{\gamma} E\left(\tilde{R}^{\gamma}\right)$$

and so, with $w_0 = 1$,

$$(16) \qquad CE = \left(E\left[\tilde{R}^{\gamma}\right]\right)^{\frac{1}{\gamma}}.$$

The certainty equivalent represents the amount of certain wealth that would be viewed with indifference to the optimal portfolio. It is computed for investors of different levels of risk aversion: low (of 0.7), medium (of 11) and high (of 31). The table shows that as risk aversion increases the value of the certainty equivalent decreases (for the unconstrained portfolio strategy as well as for the constrained). This suggests that as investors become more afraid of risk they use less risky portfolio strategies and will be expecting lower returns from those portfolios and, therefore, the certain amount of wealth they will be willing to accept with indifference will decrease.

3.2. *Results Derived for Opportunity Cost of Restricting the Number of Assets in Portfolios and of Using a Restricted VAR.*

This part of the paper discusses the results derived from calculations of the proportionate opportunity costs that reflect not only the cost of restricting the number of assets in investors' portfolios but also the cost of using a restricted VAR. In the course of calculating these costs only a *26*-asset VAR was used. Therefore, the expectations in (12) and (13) were taken over the distributions of asset returns implied by the same *26*-asset VAR for each set of available assets.

3.2.1. *Opportunity Costs*

Table 3 presents the results from calculation of 1,000 values of the proportionate opportunity cost for each of 11 different values of relative risk aversion for portfolios with three to 25 assets, based on returns distributions derived from historically occurring asset returns over the 10-year period January 1994 through December 2003.

For relative risk aversion of 0.7 the number of assets beyond which the mean value of the opportunity cost is no higher than 1% of initial wealth is 25. With 25 assets in the investor's portfolio for the risk aversion of 0.7 it is no longer helpful to make more assets available for the portfolio. As risk aversion increases, though, the well-diversified number of assets in investors' portfolios decreases. It reaches 24 assets for risk aversion of one; 23 assets for risk aversion of two; 21 assets for risk aversion of three; 11 assets for risk aversion of nine; four assets for investors with risk aversion of 29 and higher.

As Table 3 shows the highest mean of the proportionate opportunity cost, 17.6% (0.176), corresponds to the lowest level of risk aversion of 0.7 and will be incurred by an investor should he decide to invest in three assets rather than in 26 assets. The lowest mean of the proportionate opportunity cost, 0.0% (0.000), corresponds to the highest level of risk aversion of 31 and will be incurred by an investor should he decide to invest in nine or more assets rather than in 26.

The mean values of the proportionate opportunity cost of investing in four assets rather than 26 range from 16.7% for the level of risk aversion of 0.7 to 1.0% for the level of risk aversion of 31. The mean values of the proportionate opportunity cost of investing in five assets rather than 26 range from 15.9% for the level of risk aversion of 0.7 to 0.6% for the level of risk aversion of 31. The mean values of the proportionate opportunity cost

Table 3

The proportionate opportunity cost of restricting the number of assets in portfolios

and of using a restricted VAR, (θ-1), for various values of relative risk aversion

(mean values over 1,000 replications)

Number of assets in portfolios	0.7	1	2	3	9	10	11	12	29	30	31
3	0.176	0.125	0.081	0.059	0.025	0.022	0.020	0.019	0.010	0.010	0.010
4	0.167	0.120	0.076	0.055	0.020	0.018	0.017	0.017	**0.010**	**0.010**	**0.010**
5	0.159	0.114	0.069	0.050	0.018	0.017	0.016	0.015	0.008	0.008	0.006
6	0.153	0.105	0.065	0.048	0.016	0.016	0.014	0.012	0.007	0.006	0.004
7	0.147	0.094	0.060	0.046	0.014	0.013	0.012	0.011	0.005	0.004	0.003
8	0.139	0.090	0.057	0.043	0.012	0.012	0.011	0.010	0.005	0.004	0.001
9	0.132	0.084	0.053	0.039	0.011	0.011	0.010	**0.010**	0.003	0.002	0.000
10	0.125	0.075	0.048	0.037	0.010	0.010	**0.010**	0.008	0.001	0.001	0.000
11	0.119	0.070	0.046	0.034	**0.010**	**0.010**	0.007	0.007	0.000	0.000	0.000
12	0.114	0.066	0.040	0.030	0.007	0.007	0.007	0.007	0.000	0.000	0.000
13	0.104	0.058	0.036	0.027	0.006	0.007	0.006	0.006	0.000	0.000	0.000
14	0.097	0.052	0.032	0.023	0.005	0.006	0.006	0.006	0.000	0.000	0.000
15	0.090	0.046	0.028	0.021	0.005	0.005	0.005	0.005	0.000	0.000	0.000
16	0.083	0.039	0.026	0.018	0.005	0.005	0.004	0.003	0.000	0.000	0.000
17	0.077	0.036	0.021	0.016	0.003	0.004	0.004	0.003	0.000	0.000	0.000
18	0.069	0.030	0.017	0.014	0.003	0.004	0.003	0.003	0.000	0.000	0.000
19	0.062	0.025	0.015	0.012	0.003	0.003	0.003	0.001	0.000	0.000	0.000
20	0.053	0.018	0.013	0.010	0.001	0.003	0.002	0.000	0.000	0.000	0.000
21	0.045	0.015	0.011	**0.010**	0.001	0.000	0.000	0.000	0.000	0.000	0.000
22	0.035	0.013	0.010	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.028	0.011	**0.010**	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.018	**0.010**	0.005	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	**0.010**	0.005	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000

of investing in ten assets rather than 26 range from 12.5% for the level of risk aversion of

0.7 to 0.0% for the level of risk aversion of 31. It is clear that the mean values of the

proportionate opportunity cost of investing in *n* assets rather than in 26 decrease as the

number of assets available for investment increases, and as the level of relative risk

aversion increases.

This conclusion also agrees with the one drawn for Table 1. This makes sense.

When one more asset becomes available for investment, it takes an investor's portfolio to

a higher diversification level with lower unsystematic risk, and to a higher welfare level.

But the investor's marginal benefit from adding one more new asset to his portfolio decreases with every new asset. This marginal investor's benefit decreases more and more as more and more assets become available for investment. That is why as the number of assets in investors' portfolios increases, the proportionate opportunity cost of not investing in 26 assets decreases.

Conclusions drawn for Table 3 agree with those drawn for Table 1. What differs though between these two tables is the magnitude of the mean values of the proportionate opportunity cost and the well-diversified number of assets. The mean values of the proportionate opportunity cost of investing in n assets rather than in 26 are bigger for Table 3 than those for Table 1. This can be explained by the fact that the opportunity costs in Table 3 reflect not only the cost of restricting the number of assets in investors' portfolios but also the cost of using a restricted VAR in derivations of the asset returns distribution functions. In calculations of mean values of the proportionate opportunity cost for Table 1 two different VARs were used for deriving n-asset and *26*-asset probability distribution functions for every set of available assets: a VAR for n assets and a different VAR for 26 assets. In calculations of mean values of the proportionate opportunity cost for Table 3 one VAR was used for deriving n-asset and *26*-asset probability distribution functions for every set of available assets: a *26*-asset VAR. Therefore, Table 3 shows not only the cost of sub-optimal diversification but also the cost of using the restricted VAR.

As Table 3 shows higher mean values (higher than those in Table 1) of the proportionate opportunity cost of investing in n assets rather than in 26 lead to a larger well-diversified number of assets in investors' portfolios. Therefore, the result of using

the unrestricted VAR is a higher diversification benefit due to a larger number of assets in an investor's portfolio.

3.2.2. *Optimal portfolio shares*

Table 4 presents typical optimal portfolio shares for unconstrained (*26*-asset) and constrained portfolio strategies for three different levels of relative risk aversion: low (of 0.7), medium (of 11) and high (of 31). For all three levels of risk aversion there is a different well-diversified number of assets; therefore, constrained optimal portfolios for different levels of risk aversion have different numbers of assets in them. In each case illustrative optimal portfolio shares are calculated for a different set of available assets whose number is just large enough to give an opportunity cost of no more than 1.0%, the cut-off value after which the diversification benefits are assumed to be non-significant.

For Table 4 for risk aversion of 0.7 more than 100% of initial wealth, w_0, is held in the nominally risky assets (asset #1 through asset #25) as a group, and Treasury bills are held in negative quantities.

As risk aversion increases, as investors become more conservative and less risk-tolerant, the proportion of initial wealth held in Treasury bills becomes positive and increases, and correspondingly the proportion of initial wealth held in the group of nominally risky assets decreases.

The table shows that unconstrained and constrained optimal portfolio shares are not similar for different levels of risk aversion. As a matter of fact, optimal unconstrained and constrained portfolios for the low level of relative risk aversion of 0.7 have more extreme quantities (negative as well as positive) of assets than optimal unconstrained and

Table 4

Illustrative optimal portfolio shares for unconstrained and optimally constrained to include *n* assets and to use a restricted VAR portfolio strategies for different values of relative risk aversion[1]

Asset #	Relative Risk Aversion,(1-γ), equal to 0.7		Relative Risk Aversion, (1-γ), equal to 11		Relative Risk Aversion, (1-γ), equal to 31	
	Unconstrained	Constrained	Unconstrained	Constrained	Unconstrained	Constrained
1	-1.308	0.000	-0.136	0.000	-0.002	0.000
2	3.555	2.550	-0.005	0.000	0.029	0.000
3	1.127	0.764	0.109	0.024	0.223	0.054
4	0.194	0.032	-0.084	-0.017	-0.090	-0.013
5	0.584	0.402	0.218	0.000	-0.013	0.000
6	0.244	0.209	0.431	0.072	-0.015	0.000
7	-0.377	-0.328	0.023	0.000	-0.019	0.000
8	0.364	0.167	-0.005	0.000	-0.002	0.000
9	-1.794	-1.486	-0.041	-0.021	0.016	0.000
10	3.576	3.125	0.054	0.000	0.009	0.000
11	-1.105	-1.369	0.130	0.000	-0.003	0.000
12	-0.103	0.093	0.215	0.118	0.020	0.000
13	0.147	0.389	-0.085	0.000	0.029	0.000
14	1.453	1.340	0.030	0.000	0.021	0.000
15	-1.974	-1.298	0.128	0.022	-0.049	0.000
16	-2.284	-2.548	0.155	0.000	0.053	0.039
17	-0.879	-1.019	-0.007	0.000	-0.004	0.000
18	1.131	1.236	0.007	0.000	0.019	0.000
19	3.750	3.290	0.071	0.000	-0.013	0.000
20	1.979	2.258	-0.052	-0.042	0.019	0.000
21	2.026	1.897	0.121	0.078	0.007	0.000
22	6.184	6.412	0.044	0.000	0.005	0.000
23	-3.326	-1.295	-0.058	-0.031	-0.034	0.000
24	-3.322	-4.626	0.009	0.000	0.052	0.000
25	1.526	1.369	-0.022	0.000	0.081	0.000
26[2]	-10.369	-10.566	-0.250	0.797	0.663	0.919
$E(X^{*\prime}\widetilde{R})$[3]	1.341	1.324	1.026	1.011	1.010	1.001
Certainty Equivalent	1.183	1.171	1.016	1.006	1.006	0.996

[1] Numbers are not comparable across levels of risk aversion, because for each level of risk aversion a different set of available assets was used.
[2] The 26th asset is risk-free in nominal terms.
[3] Monthly gross expected returns on portfolios.

constrained portfolios for the medium level of relative risk aversion of 11 and for the high level of relative risk aversion of 31. Extremely negative quantities of assets for high risk-tolerance investors mean that the investors follow an aggressive short sale strategy.

Also Table 4 shows monthly expected returns on unconstrained and constrained optimal portfolios, $E\,(X^{*\prime}\widetilde{R})$, for the three levels of relative risk aversion. The expected returns for constrained and unconstrained optimal portfolios for risk aversion of 0.7 are very dramatic. Expected returns are of medium size for risk aversion of 11 and of small size for risk aversion of 31. Such magnitudes of expected portfolio returns for high risk-tolerance investors confirm the previously made conclusion about very aggressive short sale strategies. These magnitudes suggest very leveraged portfolios (unconstrained as well as constrained) for investors with risk aversion of 0.7. For investors with risk aversion of 11 and 31 there is, definitely, some short selling going on as well. The less aggressive short selling for medium or high risk aversion leads to lower mean portfolio returns.

Table 4 also reports the certainty equivalents calculated for the same three levels of relative risk aversion (0.7, 11 and 31). The table shows that as risk aversion increases the value of the certainty equivalent decreases (for the unconstrained portfolio strategy as well as for the constrained). This suggests that as investors become more afraid of risk they use less risky portfolio strategies and will be expecting lower portfolio returns and, therefore, the certain amount of wealth they will be willing to accept with indifference will decrease.

Conclusions drawn for Table 4 agree with those drawn for Table 2. What differs between these two tables are the magnitude of the expected portfolio returns and the

certainty equivalent values, unconstrained as well as constrained. The expected portfolio returns and the certainty equivalent values are substantially bigger in Table 4 for investors with risk aversion of 0.7, somewhat bigger for investors with risk aversion of 11, and almost the same for investors with risk aversion of 31 than those in Table 2. This can be explained by the fact that the well-diversified number of assets is larger now, Table 3, for investors of almost all levels of risk aversion than it was before, Table 1. A larger well-diversified number of assets means a higher level of diversification that offers a greater opportunity for an investor to seek higher portfolio returns. Higher expected portfolio returns will lead to higher certainty equivalents for investors.

4. *Conclusion.*

This paper investigated the opportunity cost incurred by investors when they invest in a non-well-diversified number of assets. CRRA utility functions and the proportionate opportunity cost have been used. The opportunity cost has been calculated for different values of relative risk aversion.

The analysis has been done to find (a) the effect of restricting the number of assets in investors' portfolios, and (b) the effect of restricting the number of assets and of using a restricted VAR.

My findings show that the well-diversified number of assets depends on degree of risk aversion and the way the VAR process was used in deriving the asset returns distribution functions for the purpose of evaluating the opportunity cost.

The largest well-diversified number of assets found is 25 and it is for investors with risk aversion of 0.7 in the asset returns distributions for the case with the restricted

VAR. The largest well-diversified number of assets found is 24 for investors with the same level of risk aversion but with unrestricted VAR. The lowest well-diversified number of assets found is three or less assets for investors with risk aversion 29 and higher for the restricted and unrestricted VAR cases.

For all the cases considered as the level of relative risk aversion increases the proportionate opportunity cost of investing in n assets rather than in 26 decreases, as well as the well-diversified number of assets.

There is a subtle difference though in the magnitude of the proportionate opportunity costs and in the well-diversified number of assets for the restricted and unrestricted VARs cases. For the unrestricted VAR case, as my calculations showed, the opportunity costs are lower than those for the restricted VAR case. This can be explained by the fact that the opportunity costs for the restricted VAR case reflect not only the cost of restricting the number of assets in investors' portfolios but also the cost of using a restricted VAR in derivations of the asset returns distribution functions.

Also, higher proportionate opportunity costs of investing in n assets rather than in 26 in the restricted VAR case lead to a larger well-diversified number of assets in investors' portfolios. A simple reason emerges: to compensate for the restriction and to reach the same utility level investors must invest in a larger number of assets.

Therefore, based on my calculations, I may conclude that there is definitely a welfare cost for investors to incur should they decide to invest in a non-well-diversified number of assets. This cost decreases as the non-well-diversified number of assets gets closer and closer to the well-diversified number of assets. And as far as my calculations show, only investors with very high levels of relative risk aversion (29 and above) will

incur very small costs. Those investors will place such a big proportion of their initial wealth into nominally risk-free assets that it will not matter much how many nominally risky assets they have gotten in their portfolios.

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HOW MUCH DIVERCIFICATION IS ENOUGH? –

WELFARE IMPLICATIONS FOR INVESTORS

UNDER UNCERTAINTY.

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