IS MORE ALWAYS BETTER? -

EMPIRICAL EVIDENCE ON OPTIMAL PORTFOLIO SIZE.

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RRH: IS MORE ALWAYS BETTER?

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Abstract

A restriction on the number of assets in investors' portfolios results in welfare losses for the investors. To measure these welfare losses we compare *n*-asset optimal portfolios with *26*-asset optimal portfolios by using the concept of proportionate opportunity cost along with various CRRA utility functions. The original historical asset returns data is used with a VAR in generating joint returns distributions for the portfolio formation period. We find that suboptimal diversification imposes substantial costs on investors with low levels of relative risk aversion. Investors with high levels of risk aversion incur very small or no cost at all diversifying sub-optimally. We show that investors with high levels of risk aversion place most of their initial wealth in the safe asset and, therefore, few stocks are needed to achieve optimal diversification.

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INTRODUCTION

Economists believe that portfolio choice is influenced by several factors such as risk aversion, education level, level of initial wealth, age category, borrowing constraints, family size, and gender (Bertaut, 1998; Bertaut and Haliassos, 1997; Guiso, Jappelli , and Terlizzese (1996), Guiso, Haliassos, and Jappelli (2002), Haliassos and Michaelides (2003), Jagannathan and Kocherlakota (1996), Jianakoplos and Bernasek (1998), Kennickell, Starr-McCluer, and Surrette (2000), Uhler and Cragg (1971), Ameriks and Zeldes (2000), Attanasio and Hoynes (2000)).

Empirical evidence shows that many individual investors are unsophisticated and make decisions that result in suboptimal diversification. Bernheim (1996), based on the survey of financial knowledge of Americans, points that "…[m]ost Americans are not making prudent financial decisions". In particular, many individual investors are too conservative and hold portfolios that are not well diversified (Kelly 1995, Brennan and Torous ,1999). Kelly (1995) shows that "…[t]hree quarters of the households in the top quintile (based on the survey sample) of stock ownership had fewer than ten different stocks". If individual investors hold sub-optimally diversified portfolios they may suffer large welfare losses as the result of their asset allocation decisions. Therefore, it is important to evaluate the welfare losses of suboptimal diversification. This is the question addressed herein.

Working with the mean-variance theory Fama (1972) finds that "…[m]ost of the effects of diversification … occur when the first few securities are added to the portfolio. Once the portfolio has 20 securities, further diversification has little effect". Sankaran and Patil (1999), within the same framework, find that "…[d]iversification beyond 8-10 securities may not be worthwhile". Based on this evidence, we consider an optimal unconstrained portfolio permitted to have 26[*] assets as an approximation of an infinite number of assets that gives the highest diversification gain.

To measure investors' welfare losses we compare expected utility from the optimal constrained portfolio with *n* assets with that of the unconstrained optimal portfolio (26 assets) by using the concept of opportunity cost. At a certain *n* we find that further diversification is of little help: the opportunity cost of investing in these *n* assets rather than in the unconstrained optimal portfolio does not exceed 1% of the initial wealth. Then, this number *n* will be referred to as a well-diversified number of assets.

Proportionate opportunity cost is the best way to measure investors' welfare losses because the results are readily interpretable as intuitively "large" or "small", which

[*] Number 26 was chosen arbitrary: we needed a number large enough to exceed 20 (Fama's optimal number of assets) but not too large to be manageable by a computer while running 1000 replications for different values of risk aversion.

would not be true if compensating payments were expressed in additive dollar terms. Under the assumption of the constant relative risk aversion (CRRA) utility function:

$$\text{(1)} \qquad \mathrm{U}(\tilde{w}) = \begin{cases} \dfrac{1}{\gamma}\tilde{w}^{\gamma}, \ \gamma < 1, \ \gamma \neq 0, \ \tilde{w} > 0 \\ -\infty, \ \tilde{w} \leq 0 \end{cases}$$

the proportionate opportunity cost (willingness to accept payment as a compensation for being constrained to n assets) can be calculated as (θ - 1) with θ is defined as:

$$\text{(2)} \qquad \mathrm{EU}(\theta\, w_0\, \tilde{R}_n^{optimal}) = \mathrm{EU}(w_0\, \tilde{R}_{26}^{optimal})$$

where w_0 is the initial wealth, $\tilde{R}_{26}^{optimal}$ and $\tilde{R}_n^{optimal}$ are the stochastic returns per dollar invested in 26 and n asset portfolios respectively. Solving (2) with the utility function in equation (1) yields

$$\text{(3)} \qquad \theta = \left[\frac{E(\tilde{R}_{26}^{\gamma})^{optimal}}{E(\tilde{R}_n^{\gamma})^{optimal}}\right]^{\frac{1}{\gamma}}.$$

Under the CRRA utility function specification, θ also equals to the ratio of certainty equivalents of the unconstrained and n–asset constrained optimal portfolios. Since the ratio of certainty equivalents is unitless, the proportionate opportunity cost is also unitless.

Brennan and Torous (1999) have addressed the issue of the cost of suboptimal diversification. They employ a certainty equivalent concept to measure investors' losses. The authors randomly pick starting years and the securities for the portfolios from CRSP (Center for Research of Securities Prices). To form a portfolio they use the equally-weighted portfolio rule. The authors form portfolios with different numbers of assets and calculate expected utility of holding these portfolios. The whole process of choosing a starting year, drawing securities, forming portfolios, and calculating expected utility is repeated 10,000 times. Then, for every portfolio the certainty equivalent is calculated. The certainty equivalent shows how much an investor would lose should he diversify suboptimally.

The equal-weighting rule of constructing portfolios employed by Brennan and Torous (1999) is not appealing. This rule is characterized in the literature as a "naïve" portfolio strategy (Kroll et al. (1984)). The more appealing approach, that we follow here, is to use optimal portfolio shares. Further, to generate the joint asset returns distributions we use a VAR (Vector Autoregression) to project the means of returns and to capture 120 historically occurring shocks to all asset returns. We assume that the true distribution of shocks for the investment period is given by adding those 120 sets of returns shocks with equal probabilities to the conditional means.

The procedure for calculating the proportionate opportunity cost includes random asset selection for investors' portfolios, estimation of a VAR, derivation of the joint probability distribution function of asset returns, and computation of optimal portfolios.

It is shown that with a nominally risk-free asset, the well-diversified number of assets in one's portfolio depends on the degree of risk aversion and presence of a short-selling constraint. We find that the largest well-diversified number of assets in one's portfolio is 24. As relative risk aversion (RRA) increases the well-diversified number of assets in one's portfolio decreases. It is a somewhat counterintuitive conclusion but a clear reason emerges. The well-diversified number of assets also decreases when a short-selling constraint is introduced. The results also show that for investors with high levels of RRA the well-diversified number of assets is less then three due to the fact that they place more than 90% of their initial wealth into Treasury bills.

In section 1 we describe the asset selection procedure, construct the joint probability distribution function for asset returns, and compute the optimal portfolios. Section 2 presents and discusses the results. Section 3 summarizes and concludes.

1. THE PROCEDURE

1.1 Asset Selection

Monthly data for 10,000 stocks for the period from January 1992 through December 2001 are taken from CRSP monthly database.

The procedure of calculating the proportionate opportunity cost for various n and RRA is performed 1,000 times, in each case using randomly picked nominally risky assets and Treasury bills. The entire procedure is performed both with and without a short-selling constraint. For simplicity, we will refer to the proportionate opportunity cost as the opportunity cost. In addition, we will write SSC = 1 to indicate that a short-selling constraint is present and SSC = 0 if a short-selling constraint is absent.

We compute the opportunity cost for each RRA with SSC = 1 and SSC = 0 for n=3, 4,…, 25. We start the first round by picking at random 25 nominally risky assets and setting n equal three. The unconstrained optimal portfolio will include 25 nominally risky assets and Treasury bills. The constrained optimal portfolio, with three assets, will include two nominally risky assets that we pick at random from the 25 nominally risky assets and Treasury bills. Treasury bills are risk-free only in nominal terms since inflation is uncertain in any time period. Thus, in real terms all assets are risky.

Then, to construct the optimal constrained portfolio and optimal unconstrained portfolios we obtain expected values of real returns for all risky assets and Treasury bills.

1.2. Vector Autoregressions of Returns

The nominal return on asset i at time t minus the nominal return on Treasury bills at time t gives us the excess return on asset i ($x_{i,t}$) at time t , where i=1,…,25 and t=1, …,T. Running a VAR for excess returns on the 25 assets and realized inflation

(4) $$\begin{bmatrix} x_{1,t} \\ . \\ x_{25,t} \\ \pi_t \end{bmatrix} = \begin{bmatrix} c_1 \\ . \\ c_{25} \\ c_{26} \end{bmatrix} + \begin{bmatrix} \nu_{1,1}(L) & . & . & \nu_{1,\,26}(L) \\ . & . & . & . \\ \nu_{25,1}(L) & . & . & \nu_{25,\,26}(L) \\ \nu_{26,\,1}(L) & . & . & \nu_{26,\,26}(L) \end{bmatrix} \begin{bmatrix} x_{1,t} \\ . \\ x_{25,t} \\ \pi_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ . \\ \varepsilon_{25,t} \\ \varepsilon_{\pi,t} \end{bmatrix},$$

we obtain { $\hat{c}_i$ }, { $\hat{\varepsilon}_{i,t}$ } and { $\hat{\nu}_{i,k}(L)$ }, where

(5) $$\hat{\nu}_{i,k}(L) = \hat{\delta}^1_{i,k}L^1 + \hat{\delta}^2_{i,k}L^2 + ... + \hat{\delta}^p_{i,k}L^p.$$

Then, we compute the vector of conditional expected values of excess returns and inflation as:

(6) $$\begin{bmatrix} E_T x_{1,T+1} \\ . \\ E_T x_{25,T+1} \\ E_T \pi_{T+1} \end{bmatrix} = \begin{bmatrix} \hat{c}_1 \\ . \\ \hat{c}_{25} \\ \hat{c}_{26} \end{bmatrix} + \begin{bmatrix} \hat{\nu}_{1,1}(L) & . & . & \hat{\nu}_{1,\,26}(L) \\ . & . & . & . \\ \hat{\nu}_{25,\,1}(L) & . & . & \hat{\nu}_{25,26}(L) \\ \hat{\nu}_{26,1}(L) & . & . & \hat{\nu}_{26,\,26}(L) \end{bmatrix} \begin{bmatrix} x_{1,T+1} \\ . \\ x_{25,T+1} \\ \pi_{T+1} \end{bmatrix}.$$

Next, the expected real return on asset i in period T+1, the portfolio formation period, is:

(7) $$\begin{bmatrix} E_T r_{1,T+1} \\ . \\ E_T r_{25,T+1} \end{bmatrix} = \begin{bmatrix} E_T x_{1,T+1} \\ . \\ E_T x_{25,T+1} \end{bmatrix} + \begin{bmatrix} r^n_{TB,T+1} \\ . \\ r^n_{TB,T+1} \end{bmatrix} - \begin{bmatrix} E_T \pi_{T+1} \\ . \\ E_T \pi_{T+1} \end{bmatrix}$$

where $r^n_{TB,T+1}$ is the ex ante observed nominal return on Treasury bills. The expected real return on Treasury bills in period T+1 is

(8) $$E_T r_{TB,T+1} = r^n_{TB,T+1} - E_T \pi_{T+1}.$$

Finally, the conditional probability distribution for real returns for time T+1 is determined by

(9) $$\begin{bmatrix} \tilde{r}_{1,T+1} \\ \cdot \\ \tilde{r}_{25,T+1} \\ \tilde{r}_{TB,T+1} \end{bmatrix} = \begin{bmatrix} E_T x_{1,T+1} \\ \cdot \\ E_T x_{25,T+1} \\ 0 \end{bmatrix} + \begin{bmatrix} r^n_{TB,T+1} \\ \cdot \\ r^n_{TB,T+1} \\ r^n_{TB,T+1} \end{bmatrix} - \begin{bmatrix} E_T \pi_{T+1} \\ \cdot \\ E_T \pi_{T+1} \\ E_T \pi_{T+1} \end{bmatrix} + \begin{bmatrix} \tilde{\varepsilon}_{1,T+1} \\ \cdot \\ \tilde{\varepsilon}_{25,T+1} \\ 0 \end{bmatrix} - \begin{bmatrix} \tilde{\varepsilon}_{\pi,T+1} \\ \cdot \\ \tilde{\varepsilon}_{\pi,T+1} \\ \tilde{\varepsilon}_{\pi,T+1} \end{bmatrix}$$

where $\begin{bmatrix} \tilde{\varepsilon}_{1,T+1} \\ \cdot \\ \tilde{\varepsilon}_{25,T+1} \\ \tilde{\varepsilon}_{\pi,T+1} \end{bmatrix}$ takes on the historically observed values $\begin{bmatrix} \hat{\varepsilon}_{1,t} \\ \cdot \\ \hat{\varepsilon}_{25,t} \\ \hat{\varepsilon}_{\pi,t} \end{bmatrix}$ from regression (4), t=1, 2,…, T, with equal probabilities (1/T).

This method for deriving asset returns probability distribution functions, using historically occurring innovations to asset returns captured through a VAR procedure, is superior to the VAR method mentioned in the earlier literature, e.g. Campbell and Viceira (2002). The earlier literature on derivation of asset returns probability distribution functions assumes that the distribution of asset returns is static, not evolving over time. But the reality is such that the asset returns distribution is dynamic, depending on both recent realizations and the fixed historical distribution of shocks to the dynamic asset returns process. Thus, a better way to derive asset returns probability distribution functions is to include the dynamics of the past history of asset returns.

To obtain the probability distribution of returns for the three-asset constrained portfolio the above procedure is redone using equations (4) – (9) with 26 changed to three.

1.3. Constrained Portfolios

Using the derived probability distributions for real returns we compute the constrained optimal portfolio with n=3 assets, as solution to the problem:

(10) $$\underset{\{\alpha_1,\alpha_2\}}{Max}\, EU(\widetilde{w}) = Max\ E\left\{\frac{1}{\gamma}\left[w_0\left(\alpha_1\widetilde{r}_1 + \alpha_2\widetilde{r}_2 + \left(1-\alpha_1-\alpha_2\right)\widetilde{r}_{TB}\right)\right]^{\gamma}\right\}$$

where α_1, α_2, $1 - \alpha_1 - \alpha_2$ are the portfolio shares. We find α_1, α_2 that maximize expected utility using quasi-Newton method for nonlinear optimization.

1.4. Unconstrained portfolios

The next step is to obtain the unconstrained optimal portfolio with 26 assets as the solution of the problem:

(11) $$\underset{\{\alpha_1,\ldots,\alpha_{25}\}}{Max}\, EU(\widetilde{w}) = Max\ E\left\{\frac{1}{\gamma}\left[w_0\left(\alpha_1\widetilde{r}_1 + \ldots + \alpha_{25}\widetilde{r}_{25} + \left(1-\alpha_1-\ldots-\alpha_{25}\right)\widetilde{r}_{TB}\right)\right]^{\gamma}\right\}$$

where $\alpha_1, \ldots, \alpha_{25}$ are the first 25 individual assets' portfolio shares in the unconstrained optimal portfolio. We find $\alpha_1, \ldots, \alpha_{25}$ that maximize expected utility using quasi-Newton method for nonlinear optimization.

1.5. Calculating Opportunity Cost

To calculate the opportunity cost we employ the following notation: E($\widetilde{R}_{26}^{\gamma}$)$^{\text{optimal}}$, where $\widetilde{R}_{26}$ is the gross return on the optimal unconstrained portfolio with 26 assets, and E($\widetilde{R}_{n}^{\gamma}$)$^{\text{optimal}}$, where $\widetilde{R}_n$ is the gross return on the constrained optimal portfolio with $n = 3$ assets.

E($\widetilde{R}_{26}^{\gamma}$)$^{\text{optimal}}$ (or, more completely, $E_T(\widetilde{R}_{26,T+1}^{\gamma})^{optimal}$) is determined as:

(12)

$$E_T(\widetilde{R}^{\gamma}_{26,T+1})^{optimal} = \frac{1}{T}\sum_{t=1}^{T} \left\{ \begin{bmatrix} \alpha_1^{**} & \ldots & \alpha_{25}^{**} & (1-\alpha_1^{**}-\ldots-\alpha_{25}^{**}) \end{bmatrix} \begin{bmatrix} E_T r_{1,T+1} + \varepsilon_{1,t} - \varepsilon_{\pi,t} \\ \cdot \\ E_T r_{25,T+1} + \varepsilon_{25,t} - \varepsilon_{\pi,t} \\ E_T r_{TB,T+1} + \ 0 - \varepsilon_{\pi,t} \end{bmatrix} \right\}^{\gamma}$$

where $\begin{bmatrix} \alpha_1^{**} & \ldots & \alpha_{25}^{**} & (1-\alpha_1^{**}-\ldots-\alpha_{25}^{**}) \end{bmatrix}$ is the vector of optimal unconstrained portfolio shares; and $(E_T r_{i,T+1} + \varepsilon_{i,t} - \varepsilon_{\pi,t})$ and $(E_T r_{TB,T+1} - \varepsilon_{\pi,t})$ are the possible values of real returns. The expectations are taken over the distribution implied by the 26-asset VAR.

The $E(\widetilde{R}^{\gamma}_n)^{\text{optimal}}$ is determined as follows:

$$(13) \qquad E_T(\widetilde{R}^{\gamma}_{3,T+1})^{optimal} = \frac{1}{T}\sum_{t=1}^{T} \left\{ \begin{bmatrix} \alpha_1^{*} & \alpha_2^{*} & 1-\alpha_1^{*}-\alpha_2^{*} \end{bmatrix} \begin{bmatrix} E_T r_{1,T+1} + \varepsilon_{1,t} - \varepsilon_{\pi,t} \\ E_T r_{2,T+1} + \varepsilon_{2,t} - \varepsilon_{\pi,t} \\ E_T r_{TB,T+1} + 0 - \varepsilon_{\pi,t} \end{bmatrix} \right\}^{\gamma}$$

where $\begin{bmatrix} \alpha_1^{*} \ \alpha_2^{*} \, 1-\alpha_1^{*}-\alpha_2^{*} \end{bmatrix}$ is the vector of optimal constrained portfolio shares. In equation (13) the expectations are taken over the distribution implied by the *n*-asset (in this case 3-asset) VAR. Thus, the opportunity cost will reflect exclusively the cost of restricting the number of assets.

Now, we use equation (3) to obtain the numerical value of θ. The procedure of calculating the opportunity cost for other values of *n* is similar to that of *n*=3. For each value of *n*, the procedure is repeated 1,000 times.

The above exercise is done for each of the 11 values of RRA.

1.6. Short-Selling Restriction

An optimal portfolio may require investor to hold extremely long or extremely short investment positions. These extremely long or short positions sometimes are difficult to implement in practice because investors face constraints on their portfolio holdings. For instance, Regulation T that applies to almost all investors, institutional as well as individual, requires 50% margin.

Any short-selling restriction will reduce investment opportunities for an investor. In this context it is interesting to examine how these restrictions affect the well-diversified number of assets in investor's portfolio.

The 50% margin restriction is implemented in the paper the following way. First, restrict short sales of individual assets to be no more than 50% of initial wealth:

(14) $$\alpha_i \geq -0.5\, w_0 \quad \forall i,$$

where α_i's are individual portfolio shares, and w_0 is the initial wealth normalized to 1. Then find an optimal portfolio and check if the absolute sum of all negative-valued optimal portfolio shares is less than 0.5. If it is, then calculate the opportunity cost. If the absolute sum of all negative-valued individual portfolio shares is greater than 0.5, then change the restriction on short sales of individual assets to a lower proportion of initial wealth:

(14 a) $$\alpha_i \geq -0.4\, w_0 \quad \forall i.$$

Then, find an optimal portfolio and check if the absolute sum of all negative-valued optimal portfolio shares is less than 0.5. If it is, then calculate the opportunity cost. However, if the absolute sum of all negative-valued individual portfolio shares is again grater than 0.5, then restrict short sales of individual assets to even lower proportion of initial wealth.

These reductions of the proportion of initial wealth allowed for individual shorted assets will take place until either the absolute sum of all negative-valued individual portfolio shares is not less than 0.5 or the proportion of initial wealth allowed for short positions for individual assets reaches zero. In the latter case the short-selling restriction holds automatically.

2. RESULTS

2.1. Opportunity Costs with and without a Short-Selling Constraint

Table 1 presents the results of calculation of the opportunity cost for 11 different values of RRA for portfolios with three to 25 assets with SSC=0 (SSC=0 implies that no short-selling constraint is present; SSC=1 implies that a short selling constraint is present). The 11 different values of RRA include extremely low levels (from 0.7 to 3), medium levels (from 9 to 12), and extremely high levels (from 29 to 31). Even though degrees of RRA in excess of 10 are regarded as highly unreasonable (Mehra and Prescot (1985)), it is not unthinkable that some investors might be characterized by such extreme levels of RRA.

Table 1 shows that $n = 24$ for RRA = 0.7 and n decreases as RRA increases. Observe, that n reaches 20 assets for RRA=3; nine assets for RRA=9; and three or less assets for investors with RRA greater or equal 29. Table 1 also shows that the highest opportunity cost of 16.5% corresponds to the lowest level of RRA = 0.7 and is incurred by individuals investing in three assets rather than in 26. The lowest opportunity cost, 0.0%, corresponds to the highest level of RRA = 31 and is incurred by individuals investing in seven or more assets rather than in 26. Table 1 shows that the opportunity cost of investing in n assets rather than in 26 decreases as n and RRA increase.

Table 1 should be inserted here.

Table 2 should be inserted here.

Table 2 presents the results of calculation of the opportunity cost for 11 different values of RRA for portfolios with three to 25 assets with SSC=1 and shows that $n = 11$ for RRA = 0.7 and n decreases as RRA increases. Observe, that n reaches eight assets for RRA = 3; five assets for RRA = 9; and three or less assets for investors with $\text{RRA} \geq 11$. The results indicate that the opportunity cost of 2.1% corresponds to RRA = 0.7 and will be incurred by an investor should he decide to invest in three assets rather than in 26. The opportunity cost of 0.0% corresponds to RRA = 31 and is incurred by individuals investing in five or more assets rather than in 26.

The opportunity cost of investing in four assets rather than 26 ranges from 1.9% for RRA = 0.7 to 0.2% for RRA = 31. The opportunity cost of investing in ten assets rather than 26 range from 1.0% for RRA = 0.7 to 0.0% for RRA = 31. Table 2 shows that the opportunity cost of investing in n assets rather than in 26 decreases as the number of assets available for investment increases. When one more asset becomes available for investment, this extra asset takes the portfolio to a higher diversification level that reduces idiosyncratic risk and increases investors' welfare.

The results in Tables 1 and 2 show that the opportunity cost of investing in n assets rather than in 26 decreases as RRA increases. Indeed, as RRA increases investors increase their holdings of Treasury bills (nominally risk-free asset) while decreasing their holdings of nominally risky assets. Therefore, they will need fewer stocks to achieve optimal diversification. Hence, the opportunity cost of investing in n assets rather than in 26 is lower for investors with higher RRA.

The main difference between Tables 1 and 2 is that the well-diversified number of assets, n, is lower whenever $SSC = 1$. Thus, the short-selling constraint reduces diversification benefits.

2.2. Optimal portfolio shares

Table 3 and Table 4 present typical optimal portfolio shares for unconstrained (*26*-asset) and constrained portfolio strategies for three different levels of RRA: low (RRA = 0.7), medium (RRA = 11) and high (RRA = 31). Table 3 presents optimal portfolio shares with SSC=0; Table 4 presents optimal portfolio shares with SSC=1. Note that for each RRA presented the number of assets in the optimal constrained portfolio equals n.

Table 3 shows that investors with RRA = 0.7 hold more than 100% of initial wealth in the nominally risky assets (extremely long positions in some cases) and extremely short positions in Treasury bills. As RRA increases, investors become more conservative and the proportion of initial wealth held in Treasury bills increases.

Table 3 and Table 4 should be inserted here.

In addition, Tables 3 and 4 show monthly expected returns on unconstrained and constrained optimal portfolios, $E\,(X^{*\prime}\tilde{R})$. The net expected portfolio return (gross expected monthly portfolio return minus 1.0, multiplied by 100%) for constrained and unconstrained optimal portfolios with RRA = 0.7 and SSC=1 is very large. Expected returns are average for RRA = 11 and small for RRA = 31. High magnitudes of expected portfolio returns for investors with low RRA are due to their aggressive short selling strategies.

Tables 3 and 4 also report the certainty equivalents calculated for the three levels of RRA. The ratio of certainty equivalents is another measure of the opportunity cost (Brennan and Torous (1999)).

The certainty equivalent (CE) is defined by

$$\frac{1}{\gamma}\,CE^{\gamma} = \frac{1}{\gamma}\,w_0^{\gamma}\,E\left(\widetilde{R}^{\gamma}\right) \tag{15}$$

with $w_0 = 1$,

$$CE = \left(E\left[\widetilde{R}^{\gamma}\right]\right)^{\frac{1}{\gamma}}. \tag{16}$$

The certainty equivalent represents the amount of certain wealth that would be viewed with indifference relative to the expected optimal portfolio return. Tables 3 and 4 show that as RRA increases CE decreases. As investors become more risk averse they employ safer portfolio strategies (less or no short-selling and/or larger holding of Treasury bills) and expect lower returns. Hence, the amount of certain wealth they are willing to accept in exchange for lower expected portfolio return is lower.

3. CONCLUSION

In this paper we have investigated the opportunity cost incurred by investors when they diversify suboptimally by using CRRA utility functions and the proportionate opportunity cost. The opportunity cost has been calculated for different values of relative risk aversion (including extreme levels of relative risk aversion) with and without a short-selling constraint.

We have found that the well-diversified number of assets depends on degree of risk aversion and presence of the short-selling constraint. The largest well-diversified number of assets found is 24 for investors with risk aversion of 0.7 without the short-selling constraint. The lowest well-diversified number of assets found is three or less assets for investors with risk aversion 29 and higher with the short-selling constraint. As the level of relative risk aversion increases both the proportionate opportunity cost of investing in n assets rather than in 26 and the well-diversified number of assets decrease.

We have shown that there is definitely a cost for investors to incur should they decide to diversify suboptimally. The cost decreases as the number of assets gets closer to the well-diversified number of assets. Only investors with very high levels of relative risk aversion (29 and above) will incur very small costs. Those investors will place such a big proportion of their initial wealth into nominally risk-free assets that it will not matter much how many nominally risky assets they have in their portfolios.

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Table 1

The proportionate opportunity cost of restricting the number of assets in portfolios, (θ-1), for various values of RRA with SSC=0[1]

Number of assets in portfolios	0.7	1	2	3	9	10	11	12	29	30	31
3	0.165	0.118	0.062	0.041	0.013	0.012	0.012	0.011	**0.006**	**0.005**	**0.004**
4	0.159	0.092	0.061	0.039	0.012	0.012	0.011	0.011	0.005	0.003	0.002
5	0.157	0.091	0.058	0.038	0.012	0.011	0.011	0.011	0.005	0.003	0.001
6	0.147	0.084	0.056	0.036	0.011	0.011	0.011	**0.010**	0.003	0.001	0.001
7	0.141	0.081	0.053	0.034	0.011	0.011	**0.010**	0.007	0.003	0.001	0.000
8	0.135	0.080	0.051	0.032	0.011	**0.010**	0.008	0.007	0.001	0.000	0.000
9	0.126	0.078	0.049	0.030	**0.010**	0.009	0.007	0.006	0.001	0.000	0.000
10	0.119	0.077	0.045	0.029	0.009	0.008	0.007	0.005	0.000	0.000	0.000
11	0.113	0.075	0.042	0.027	0.008	0.007	0.005	0.005	0.000	0.000	0.000
12	0.105	0.073	0.040	0.026	0.007	0.007	0.005	0.005	0.000	0.000	0.000
13	0.098	0.069	0.037	0.024	0.007	0.006	0.005	0.003	0.000	0.000	0.000
14	0.090	0.063	0.034	0.023	0.006	0.005	0.003	0.001	0.000	0.000	0.000
15	0.083	0.057	0.031	0.020	0.006	0.005	0.003	0.001	0.000	0.000	0.000
16	0.076	0.054	0.029	0.019	0.006	0.005	0.003	0.001	0.000	0.000	0.000
17	0.071	0.051	0.027	0.017	0.005	0.005	0.001	0.000	0.000	0.000	0.000
18	0.063	0.044	0.024	0.015	0.005	0.004	0.001	0.000	0.000	0.000	0.000
19	0.054	0.040	0.021	0.011	0.005	0.003	0.000	0.000	0.000	0.000	0.000
20	0.047	0.035	0.018	**0.010**	0.004	0.003	0.000	0.000	0.000	0.000	0.000
21	0.037	0.028	0.012	0.007	0.003	0.000	0.000	0.000	0.000	0.000	0.000
22	0.028	0.018	**0.010**	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.019	**0.010**	0.006	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	**0.010**	0.006	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.004	0.030	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

[1]SSC=0: there is not short-selling constraint.

Table 2

The proportionate opportunity cost of restricting the number of assets in portfolios, (θ-1), for various values of RRA with SSC=1[1]

Number of Assets in portfolios	0.7	1	2	3	9	10	11	12	29	30	31
3	0.021	0.019	0.017	0.016	0.011	0.010	**0.010**	**0.010**	**0.005**	**0.005**	**0.004**
4	0.019	0.018	0.016	0.014	0.010	**0.010**	0.009	0.008	0.003	0.003	0.002
5	0.017	0.016	0.014	0.012	**0.010**	0.009	0.008	0.006	0.001	0.001	0.000
6	0.016	0.015	0.011	0.011	0.009	0.007	0.006	0.004	0.000	0.000	0.000
7	0.014	0.012	0.011	0.011	0.008	0.006	0.004	0.002	0.000	0.000	0.000
8	0.013	0.011	0.010	**0.010**	0.006	0.004	0.002	0.001	0.000	0.000	0.000
9	0.011	0.010	**0.010**	0.009	0.005	0.002	0.001	0.000	0.000	0.000	0.000
10	0.010	**0.010**	0.009	0.008	0.004	0.001	0.000	0.000	0.000	0.000	0.000
11	**0.010**	0.009	0.008	0.007	0.002	0.001	0.000	0.000	0.000	0.000	0.000
12	0.009	0.008	0.007	0.006	0.001	0.000	0.000	0.000	0.000	0.000	0.000
13	0.008	0.007	0.006	0.005	0.001	0.000	0.000	0.000	0.000	0.000	0.000
14	0.007	0.006	0.005	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.006	0.006	0.005	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16	0.005	0.005	0.005	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	0.005	0.005	0.004	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.005	0.004	0.003	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000
19	0.004	0.004	0.003	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.004	0.003	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21	0.004	0.003	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.004	0.003	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.003	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

[1] SSC=1: short-selling constraint is present.

Table 3

Illustrative optimal portfolio shares for unconstrained and optimally constrained to include *n* assets portfolio strategies for different values of RRA[1] with SSC=0[4]

Asset #	RRA = 0.7		RRA = 11		RRA = 31	
	Unconstrained	Constrained	Unconstrained	Constrained	Unconstrained	Constrained
1	3.055	2.764	0.049	0.000	0.001	0.000
2	-0.081	-0.129	0.024	0.006	0.049	0.023
3	0.337	0.000	0.076	0.000	-0.081	0.000
4	2.621	2.145	-1.000	0.000	0.016	0.000
5	-0.427	-0.502	0.066	0.000	0.166	0.088
6	0.149	0.008	0.195	0.000	0.001	0.000
7	0.638	0.664	0.031	0.182	0.035	0.000
8	-0.019	-0.385	-0.026	0.000	0.049	0.000
9	-1.991	-2.197	-0.014	0.000	0.008	0.000
10	-0.305	-0.309	0.049	0.000	0.018	0.000
11	0.647	0.544	-0.042	0.000	0.066	0.000
12	1.004	1.191	0.097	0.089	-0.029	0.000
13	1.456	1.515	0.007	0.002	0.016	0.000
14	0.119	0.095	-0.082	0.000	-0.007	0.000
15	-0.572	-0.396	0.201	0.191	-0.049	0.000
16	0.137	0.289	0.053	0.000	-0.082	0.000
17	-1.837	-1.827	0.091	0.000	0.021	0.000
18	0.001	0.172	-0.025	0.000	0.005	0.000
19	0.266	0.294	0.059	0.000	0.018	0.000
20	-0.997	0.000	0.022	0.000	-0.003	0.000
21	0.294	0.256	-0.149	-0.059	-0.009	0.000
22	0.479	0.574	0.086	0.000	0.031	0.000
23	0.551	0.661	-0.046	0.000	-0.016	0.000
24	0.430	0.367	0.042	0.000	0.021	0.000
25	1.497	1.257	0.070	0.000	0.024	0.000
26[2]	-6.451	-6.051	0.265	0.589	0.736	0.889
$E(X^{*\prime}\tilde{R})$[3]	1.166	1.156	1.018	1.007	1.007	1.000
Certainty Equivalent	1.088	1.077	1.009	0.999	1.001	0.993

[1] *Numbers are not comparable across levels of risk aversion, because for each level of risk aversion a different set of available assets was used.*
[2] *The 26th asset is risk-free in nominal terms.*
[3] *Monthly gross expected returns on portfolios.*

[4] *SSC=0: there is no short-selling constraint*

Table 4

Illustrative optimal portfolio shares for unconstrained and optimally constrained to include *n* assets portfolio strategies for different values of RRA[1] with SSC=1[4]

Asset #	RRA = 0.7		RRA = 11		RRA = 31	
	Unconstrained	Constrained	Unconstrained	Constrained	Unconstrained	Constrained
1	-0.010	0.000	-0.020	0.000	-0.020	0.000
2	-0.030	-0.055	-0.010	0.000	0.009	0.000
3	-0.020	0.000	0.176	0.000	0.065	0.000
4	-0.020	0.000	-0.020	0.000	-0.010	0.000
5	-0.010	0.000	0.009	0.000	0.026	0.000
6	-0.030	0.000	0.237	0.019	0.001	0.000
7	-0.020	0.000	0.019	0.000	-0.009	0.000
8	-0.020	0.000	-0.020	0.012	-0.020	0.000
9	-0.050	-0.055	0.015	0.000	-0.020	0.000
10	0.377	0.602	0.026	0.000	0.012	0.001
11	-0.010	0.000	-0.020	0.000	-0.006	0.000
12	-0.010	0.055	-0.020	0.000	-0.020	0.000
13	-0.010	0.000	0.061	0.000	0.139	0.003
14	-0.030	0.000	0.087	0.000	-0.020	0.000
15	-0.010	-0.055	0.024	0.000	-0.001	0.000
16	0.281	0.152	0.029	0.000	0.059	0.000
17	-0.030	-0.055	-0.020	0.000	-0.020	0.000
18	-0.010	0.000	-0.020	0.000	0.041	0.000
19	-0.040	-0.055	-0.020	0.000	0.012	0.000
20	-0.010	0.000	-0.020	0.000	-0.020	0.000
21	-0.010	0.000	-0.020	0.000	0.008	0.000
22	-0.020	0.000	-0.020	0.000	0.001	0.000
23	-0.020	0.000	-0.019	0.000	-0.005	0.000
24	-0.030	-0.055	0.042	0.000	-0.002	0.000
25	0.802	0.686	0.181	0.000	0.018	0.000
26[2]	-0.060	-0.115	0.331	0.969	0.769	0.996
$E(X^{*\prime}\tilde{R})$[3]	1.056	1.042	1.017	1.006	1.006	1.000
Certainty Equivalent	1.047	1.036	1.009	0.999	1.004	0.994

[1] *Numbers are not comparable across levels of risk aversion, because for each level of risk aversion a different set of available assets was used.*
[2] *The 13th asset is risk-free in nominal terms.*
[3] Monthly gross expected returns on portfolios.

[4] *SSC=1: short-selling constraint is present*